Fluid Mechanics

The atmosphere is a fluid!

Some definitions

- A fluid is any substance which can flow
  - Liquids, gases, and plasmas
- Fluid statics studies fluids in equilibrium
  - Density, pressure, buoyancy
- Fluid dynamics studies fluids in motion
  - Extremely complex topic, just lightly covered

Density

- An *intensive* property of a substance
- May vary with temperature or pressure
- A *homogenous* material has the same density throughout
- For a substance of mass m and volume V

$$\rho = m/V$$

where the Greek letter ρ (rho) is density

Atmospheric Density

- Because the atmosphere is a gas, and gases are compressible, density will vary with height
- The atmosphere will be more dense at the surface and less dense as altitude increases

$$\rho_{air} = 1.20\ kg/m^3 \text{ @ } 1\ atm \text{ and } 20\ ^{o}C$$

Pressure in a fluid

- Pressure is the ratio of the perpendicular force applied to an object and the surface area to which the force was applied

$$P = F/A$$

- Don’t confuse pressure (a scalar) with force (a vector).

The Pascal

- The SI unit of pressure is the pascal (Pa)

$$1 \text{ Pa} = 1 \text{ N/m}^2$$

  named after Blaise Pascal (1623-1662). One pascal also equals 0.01 millibar or 0.00001 bar.
- Meteorologists have used the millibar as a unit of air pressure since 1929.

Millibar or hPa?

- When the change to scientific units occurred in the 1960's many meteorologists preferred to keep using the magnitude they were used to and use a prefix "hecto" (h), meaning 100.
- Therefore, 1 hectopascal (hPa) = 100 Pa = 1 millibar (mb). 100,000 Pa equals 1000 hPa which equals 1000 mb. The units we refer to in meteorology may be different, however, their numerical value remains the same.

Atmospheric Pressure

- We live at the bottom of a sea of air. The pressure varies with temperature, altitude, and other weather conditions
- The average at sea level is 1 atm (atmosphere)
- Some common units used:

1 atm = 101,325 Pa

1 atm = 1013.25 mb = 1013.25 hPa

1 atm = 760 mmHg = 29.96 inHg

1 atm = 14.7 lb/in^2

Atmospheric Layer Boundaries

- The layers of our atmosphere can vary in thickness from the equator to the poles
- The layer boundaries occur at changes in temperature profile

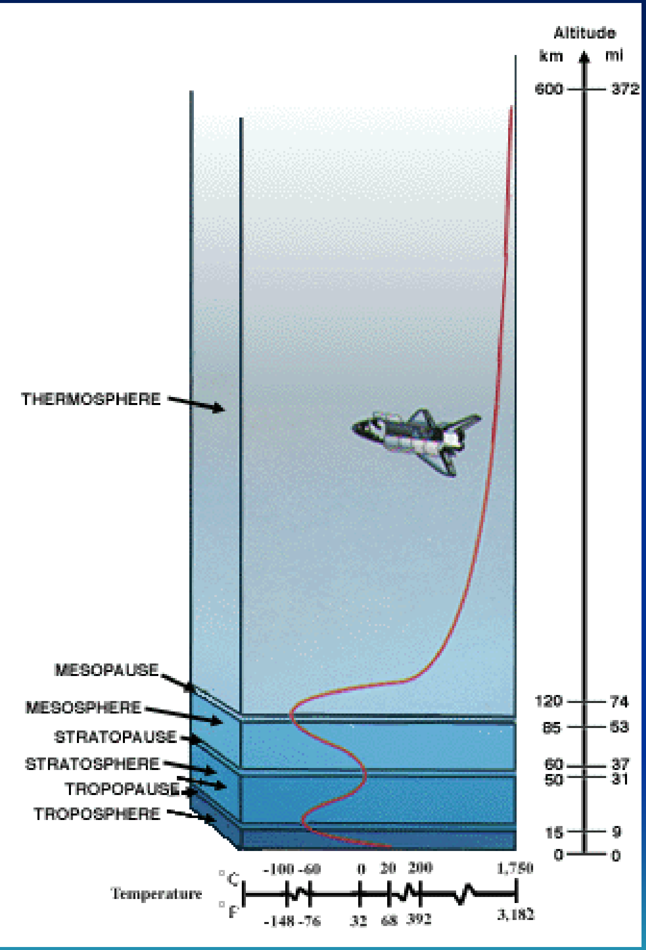
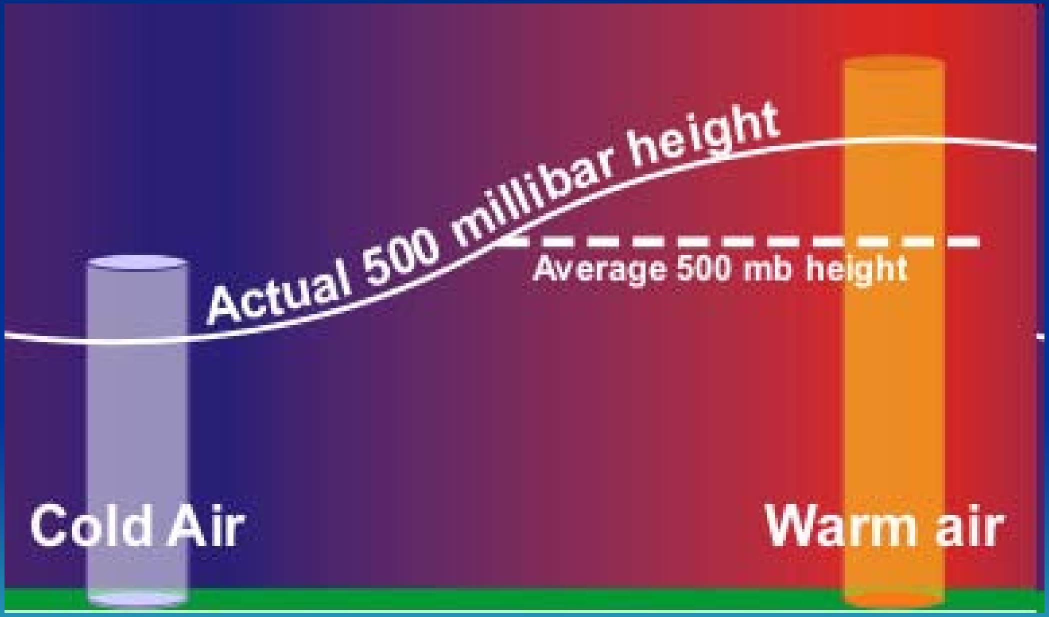


Image courtesy NASA

Temperature affects pressure

- When air warms, it expands, becoming less dense. Lower density means a volume of air weighs less, therefore applying less pressure.



← Approx 5.6 km

Image Courtesy NOAA

Fluid Pressure

- A column of fluid h = 4 m high will exert a greater pressure than a column h = 2 m
- What will the pressure be due to this fluid?

Force = mg   Area = A

But $m = \rho V$ and $A = V/h$

$$P_{fluid} = \rho Vg/V/h = \rho gh$$

Assuming uniform density for the fluid

$P_{fluid} = \rho g h$

- The pressure due to a fluid depends only on the average density and the height
- It does not depend on the shape of the container!
- The total pressure at the bottom of an open container will be the sum of this fluid pressure and the atmospheric pressure above

$$P = P_0 + P_{fluid} = P_0 + \rho g h$$

Compressibility

- Liquids are nearly incompressible, so they exhibit nearly uniform density over a wide range of heights (ρ only varies by a few percent)
- Gases, on the other hand, are highly compressible, and exhibit significant change in density over height

ρ_{air} at sea level ~ $3\rho_{air}$ at Mt Everest's peak

Pascal's Law

- Pressure applied to a contained fluid is transmitted undiminished to the entire fluid and to the walls of the container
- You use this principle to get toothpaste out of the tube (squeezing anywhere will transmit the pressure throughout the tube)
- Your mechanic uses this principle to raise your car with a hydraulic lift

Absolute and Gauge Pressure

- Your tire maker recommends filling your tires to 30 psi. This is in addition to the atmospheric pressure of 14.7 psi (typical)
- Since $P = P_0 + P_{fluid}$, the absolute pressure is P, and the gauge pressure is P_{fluid}
- In this case, the gauge pressure would be 30 psi and the absolute pressure would be 44.7 psi

(psi = pounds per square inch)

Measuring Pressure

- There are two main types of instruments used to measure fluid pressure
- The Manometer
  - Blood pressure is measured with a variant called the sphygmomanometer (say that three times fast!)
- The Barometer
  - Many forms exist

The Manometer

- Open-tube manometer has a known pressure P_0 enclosed on one end and open at the other end

$$P_0 - P_a = \rho g h$$

- If $P_a > P_0$, the fluid will be forced toward the closed end (h is neg, as shown)

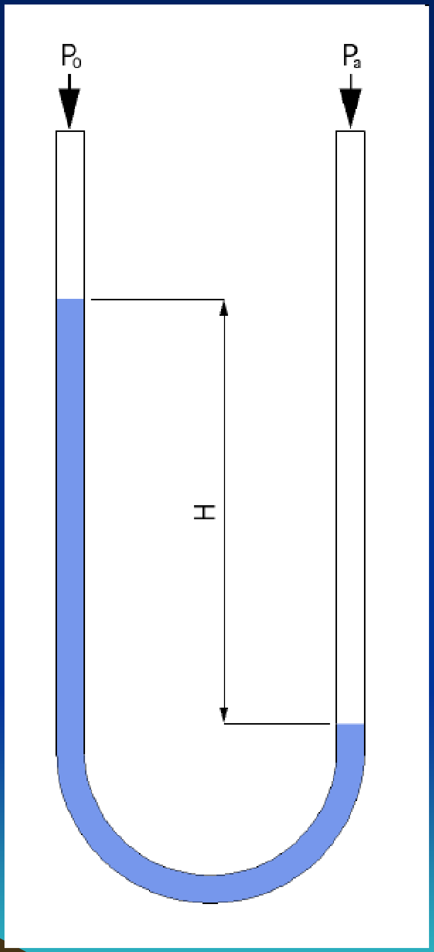


Image from Ruben Castelnuovo - Wikipedia

The Barometer

- Filling a tube (closed at one end) with liquid, then inverting it in a dish of that liquid
- A near vacuum will form at the top
- Since $P_{fluid} = \rho gh$, the column of liquid will be in equilibrium when $P_{fluid} = P_{air}$
- Meteorologist speak of 29.96 inches. This is the height of a column of Mercury which could be supported by that air pressure

Superman and the straw

- Could Superman sit at the top of the Empire State Building and sip a beverage at the ground floor through a long straw?
- No! It is the air pressure on the top of the beverage which pushes the liquid up through the straw, regardless of what you do at the other end

Max height at 1 atm = $\rho g h$ = 10.34 m

assuming ρ_{water} = 1000 kg/m^3 and g = 9.8 m/s^2

How do they get water up there?

- You can't leave it to atmospheric pressure to raise water above ~ 34 feet (10.34 m), so how do they get water to the top of the Empire State Building (381 m)?
- You must have a closed system (water pipes), then use Pascal's Law to apply pressure evenly throughout the pipes

Buoyancy

- Archimedes's principle states that the buoyant force on an object immersed in a fluid is equal to the weight of the fluid displaced by that object

Volume of object = V_O

Weight of fluid = $m_F g$

$m_F = \rho_F V_O$

$F_B = \rho_F V_O g$

Lighter than air?

- The atmosphere is a fluid
- Scientific instruments are carried into the atmosphere by balloons filled with helium
- At some point, $\rho_{air} = \rho_{balloon}$ and the balloon will stop rising (F_B up = F_G down)
- This is why there is little *in situ* scientific data regarding the Mesosphere – it is above the height of a balloon's buoyancy, but still too much air for satellite orbits (meteors burn up here!)

Fluid Dynamics

- Just as there is a concept of an ideal gas, there is a notion of an ideal fluid
  - Not compressible
  - No internal friction (viscosity)
- Liquids approximate ideal fluids
- Gases are close to ideal only if the pressure differences from one area to another are small

Fluid Flow

- Fluid flow can be
  - *Laminar* (smooth, sheet-like)
  - *Turbulent* (chaotic, eddy circulation)
- A *streamline* is a curve which traces the fluid flow (points are tangent to velocity)
- For an Airfoil and Curveball simulator, go to the NASA site http://www.grc.nasa.gov/WWW/K-12/FoilSim/index.html

Continuity Equation

- Given an ideal fluid (non-compressible) in a pipeline or other conduit (no sources or sinks), whatever flows in must flow out
- This is a statement of conservation of mass – whatever flows across one boundary must arrive at the next boundary

$$\rho A_1 v_1 \Delta t = \rho A_2 v_2 \Delta t$$

$$A_1 v_1 = A_2 v_2$$

Pressure in a moving fluid

- As shown by the continuity equation, the speed of flow may vary along a given path
- We can derive the Bernoulli equation which relates pressure, flow speed, and height for an ideal fluid
- The derivation is complex, involving the application of the work-energy principle and conservation of energy

Bernoulli's equation

- The result is Bernoulli's equation:

$$P_1 + \rho g y_1 + 1/2\ \rho v_1^2 = P_2 + \rho g y_2 + 1/2\ \rho v_2^2$$

- The subscripts refer to any two points along the fluid
- At rest ($v_1 = v_2 = 0$), this reduces to the static equation
- This is ONLY valid for ideal fluids!

Fluid speed and pressure

- The result: in a pipe of varying diameter, the fluid will always flow faster in the narrower pipe
- The pressure will be smaller when the flow is faster

Fluid Resistance and Drag

- At low speed, *fluid resistance* is proportional to velocity

$$F = kv$$

- At higher speeds, *air drag* is proportional to the square of velocity

$$F = Dv^2$$

- Where k and D are constants of proportionality which depend on the size and shape of the object and the properties of the fluid
- Can’t use kinematic equations with constant aceleration to solve for motion!