

# The IMF Mapping Functions

A. Niell

August 15, 2002

Rev 1 Section 4.1

November 7, 2002

Rev 2 Section 4.2

May 5, 2003

## 1. Introduction

The IMF hydrostatic and wet mapping functions make use of the output of a numerical weather model (NWM) to provide the *in situ* information needed to provide the state of the atmosphere at the time of observations. The design goals of the derivation of the IMF mapping function were to avoid making multiple raytrace calculations at each grid point of the numerical weather model, and to obtain a single parameter for each of the hydrostatic and wet components to be used with the geographic location to calculate the mapping function. For the hydrostatic mapping function the use of a single quantity from the NWM, the 200 hPa geopotential height, provides the required parameter. From the geopotential heights around a site both the azimuthally symmetric mapping function and the hydrostatic gradient can be calculated. For the wet mapping function, since it is not in hydrostatic equilibrium, the vertical distribution of refractivity due to water vapor must be utilized. The adopted parameter reflects this distribution as well as the changing geometry with height above surface due to the curvature of the Earth. This parameter is given by the ratio of {the line-of-site delay along a ray path at beginning elevation of 3.3°} to {the zenith wet delay}. For both the hydrostatic and wet mapping functions the adopted parameter is one input, along with the geographic location of the site, from which the constants of the three-term continued fraction in  $1/\sin(\text{elevation})$  are calculated:

$$m(\mathbf{e}) = \frac{1 + \frac{a}{1 + \frac{b}{1 + c}}}{\sin(\mathbf{e}) + \frac{a}{\sin(\mathbf{e}) + \frac{b}{\sin(\mathbf{e}) + c}}}$$

where  $\mathbf{e}$  is the *vacuum* elevation of the incoming ray.

In this note I will describe the use of the 200 hPa geopotential heights for the hydrostatic mapping function, give the algorithm for calculating the input parameter for the wet mapping function, and provide the equations for calculating the parameters  $a$ ,  $b$ ,  $c$  of the continued fraction for both the hydrostatic and wet mapping functions.

## 2. Azimuthally symmetric hydrostatic mapping function

The mapping function,  $m(\mathbf{e})$ , is defined as the ratio of the electrical path length (also referred to as the delay) through the atmosphere at geometric elevation,  $e$ , to the electrical path length in the zenith direction. For a planar atmosphere the ratio would be given by

$1/\sin(\epsilon)$ . As the ratio of the thickness of the atmosphere to the radius of the earth decreases, the atmosphere appears more planar. Thus a possible proxy for the mapping function is some quantity that is a measure of the thickness of the atmosphere.

The zenith hydrostatic delay is proportional to the integral of the density of the hydrostatic part of the atmosphere. Thus, because the atmosphere is very close to hydrostatic equilibrium, a contour of constant pressure (isobar) provides the height above the geoid of a constant delay, and these heights might be expected to serve as the parameter needed for the hydrostatic mapping function.

The geopotential heights from a numerical weather model were compared with hydrostatic mapping functions at  $5^\circ$  for twenty-eight sites for the year 1992. The height of the 200 hPa level ( $z_{200}$ ) showed the highest correlation with the  $5^\circ$  hydrostatic mapping function, making it the best candidate for the proxy parameter. Fortunately, the 200 hPa level was widely available for use by the global community at the time that the coefficients of the hydrostatic mapping function were being evaluated. (*For the paper a figure will be included to show the agreement of the 200 hPa level and  $mf(5^\circ)$  for the 1992 data.*)

The following procedure was used to determine the coefficients,  $a$ ,  $b$ ,  $c$ , of the continued fraction. All of the profiles ( $\sim 730$ ) for each of the twenty-eight sites were raytraced for nine elevations from  $90^\circ$  down to  $3^\circ$ . For each profile the coefficients  $a$ ,  $b$ , and  $c$  that best fit the ratios of the hydrostatic component of the delays at the nine elevations to the zenith delay were estimated by least squares. From the mean geopotential height over the year for each site a reference geopotential surface that is a function of latitude of the form  $\cos(2 \cdot \text{latitude})$  was determined. A latitude dependence is expected since the temperature profile that results in the same pressure and geopotential height at high latitudes will be quite different from the corresponding profile at the equator. The functional form  $\cos(2 \cdot \text{latitude})$  is empirical and is suggested by symmetry about the axis of rotation and about the equator. Each of the coefficients was then expanded to second order in  $z_{200} - z_{200, \text{ref}}$  and  $\cos(2 \cdot \text{latitude})$ . For the  $a$  coefficient, all but the  $z_{200}^2$  term were found to be significant. For  $b$  only the mean value is used, and  $c$  is linear in  $\cos(2 \cdot \text{latitude})$ . This hydrostatic mapping function is referred to as IMFh.

The coefficients are an expansion in the parameters  $z_{200}$  and  $\cos(2 \cdot \text{latitude})$

$$\begin{aligned} a &= a_{\text{lat}} + d_{\text{adzlat}} \cdot (z_{200} - z_{\text{ref}}) \\ b &= b_{\text{m0}} \\ c &= c_{\text{m0}} + c_{\text{m1}} \cdot \cos(\text{latitude} - \text{latc0}) \end{aligned}$$

where

$$\begin{aligned} a_{\text{lat}} &= a_{00} + a_{01} \cdot \cos(2 \cdot (\text{latitude} - \text{lata0})) \\ d_{\text{adzlat}} &= (d_{\text{adz0}} + d_{\text{adz1}} \cdot \cos(2 \cdot (\text{latitude} - \text{latd0}))) \\ z_{\text{ref}} &= z_0 + z_1 \cdot \cos(2 \cdot (\text{latitude} - \text{latz0})) \end{aligned}$$

and the constants are given by

$$\begin{aligned} a_{00} &= 0.00124; \\ a_{01} &= 4.e-5 ; \\ \text{lata0} &= 2.0 ; \end{aligned}$$

```
dadz0 = 7.4e-8 ;
dadz1 = -1.6e-8 ;
latd0 = 0.0 ;
bm0 = 0.002905;
cm0 = 0.0634 ;
cm1 = 0.0014 ;
latc0 = 0.0 ;
z0 = 11836.0 ;
z1 = 619.0 ;
latz0 = 3.0;
```

Note that some constants are set to zero, but they are retained for generality.

For the hydrostatic mapping function a correction must be made for the height of the site above sea level (see Niell 1996 or include here). The coefficients below are the same as for NMF but have been confirmed for higher initial site heights (up to 8 km above sea level).

```
a_ht = 2.53e-5;
b_ht = 5.49e-3;
c_ht = 1.14e-3;
```

### 3. Hydrostatic gradient mapping function

If the hydrostatic gradient can be removed, estimation of the remaining gradient will more closely reflect the wet gradient. In the same way that the 200 hPa surface serves as a proxy for the parameterization of the hydrostatic mapping function, I have modeled the gradient of the 200 hPa isobaric surface as representative of the hydrostatic gradient. The effect of the gradient is calculated by altering the apparent zenith direction from which the elevation of the hydrostatic mapping function is measured. The altered zenith is the the direction of the normal to the isobaric surface.

The input for IMFh is the 200 hPa geopotential height above the site. This height can be obtained from, for example, the gridded output from a numerical weather model. Tests on the CONT94 data using the GOES-1 analysis from the DAO showed that the expected improvement in vertical estimation was achieved for this limited data set (Niell 2001). The same data can be used to calculate the gradient of the 200 hPa isobaric surface above each site.

#### 3.1. Model for the hydrostatic gradient

At 3° elevation the 200 hPa surface is at a horizontal distance of approximately 200 km. The 2° latitude by 2.5° longitude grid of the DAO re-analysis data corresponds to a spacing of 200 km by 300 km near the equator. Although the atmosphere is curved over these distances (this is the reason the mapping function decreases less rapidly than  $1/\sin(e)$ ), the geopotential heights over a small area may be approximated as planar. The normal to this plane is then the tilt of the hydrostatic component of the atmosphere.

Three parameters must be estimated from the geopotential heights at the grid points of the NWM: the geopotential height above the site and the two components of the gradient. Define the following vectors:

$\mathbf{x}_0$  = position of geopotential surface above site

$\mathbf{w}_m$  = position of geopotential surface at grid points

$\mathbf{h}$  = normal to the plane that is fit to geopotential heights at grid points

The components of  $\mathbf{x}_0$  are  $(x_0, y_0, z_0)$ .  $x_0$  and  $y_0$  are the horizontal coordinates of the site, and  $z_0$ , the 200 hPa height above the site, is to be estimated. The x and y components of  $\mathbf{w}_m$  are the horizontal coordinates of the grid points. The z-components of the  $\mathbf{w}_m$  are the geopotential heights at the grid points. The gradient direction is obtained as the zenith angle and azimuth of the normal to the plane containing the geopotential heights. The estimated plane must satisfy the equations

$$\mathbf{h} \cdot (\mathbf{w}_m - \mathbf{x}_0) = 0$$

The z-component of this equation, after dividing by  $h_z$ , is

$$w_{mz} = z_0 - \frac{h_x}{h_z}(w_{mx} - x_{0x}) - \frac{h_y}{h_z}(w_{my} - x_{0y})$$

Only the ratios of the components of the normal vector need be estimated since it is a unit vector. So the vector of unknowns is  $\mathbf{y} = (z_0, h_{xz}, h_{yz})$ , where  $h_{xz} = h_x/h_z$ , and the vector of observables,  $\mathbf{w}_{mz}$ , is composed of the geopotential heights at the grid points to be used for estimating the plane.

Although the uncertainties of the observables are likely to be the same for all points, it is useful to include them in the estimation. Schubert et al (1993) found a standard deviation of approximately 20 m in the differences of the geopotential heights between analyses by the Goddard Space Flight Center Data Assimilation Office (DAO) and the European Center for Medium Range Weather Forecasting (ECMWF). This value can be used as the uncertainty of the measurements in order to provide an estimate of the uncertainty of the gradient or to evaluate how well the geopotential heights are modeled by a plane.

The parameters are estimated using weighted least-squares.

$$\mathbf{y} = (\mathbf{A}^T \mathbf{W} \mathbf{A})^{-1} \mathbf{A}^T \mathbf{W} \mathbf{w}_{mz}$$

where  $\mathbf{A}$  is the partials matrix with row elements of  $(1 \quad -(w_{mx}-x_{0x}) \quad -(w_{my}-x_{0y}))$ , and  $\mathbf{W}$  is the weight matrix whose diagonal elements are the reciprocal of the square of the uncertainty in the geopotential height,  $\sigma_0$ . The off-diagonal elements are taken to be zero, i.e. the geopotential heights are assumed to be uncorrelated. If the geopotential heights are positively correlated, the uncertainty of  $z_0$ , the value of z200 above the site, will be underestimated and the apparent goodness of fit will be optimistic. However, even for the full uncertainty of 20m, the uncertainty in the delay is less than 3 mm at 5°, corresponding to less than 1 mm of height error.

Since the gradient vector is a unit vector giving the direction of the normal to the plane defined by the isobaric surface,

$$h_z = (1 + r_{xy}^2)^{-1/2}$$

where

$$r_{xy} = (h_{xz}^2 + h_{yz}^2)^{1/2}$$

As implemented in *matlab*, the azimuth of the gradient, measured east from north, is given in degrees by

$$\lambda = 180^\circ/\pi * \text{atan2}(h_{xz}, h_{yz}).$$

The elevation angle of the gradient is given in degrees by

$$\epsilon = 180^\circ/\pi * \text{atan2}(1, r_{xy}).$$

The zenith angle of the gradient is given in degrees by

$$\theta = 90^\circ - \epsilon.$$

### 3.2. Uncertainties in azimuth and elevation

Although the estimation of the gradient uses rectangular components,  $h_{xz}$  and  $h_{yz}$ , a more physical system is the zenith angle (or elevation angle) and azimuth of the gradient. In order to make use of this representation both the estimated direction and uncertainties must be transformed.

Azimuth ( $\lambda$ ):

$$\begin{aligned} \tan(\lambda) &= h_{xz}/h_{yz} \\ \sigma_\lambda^2 &= (1/r_{xy})^2 * (\cos^2(\lambda) * \sigma_{xz}^2 + \sin^2(\lambda) * \sigma_{yz}^2 \\ &\quad - 2\cos(\lambda)\sin(\lambda) \langle \sigma_{xz}\sigma_{yz} \rangle) \end{aligned}$$

Elevation ( $\epsilon$ ):

$$\begin{aligned} \tan(\epsilon) &= 1/r_{xy} \text{ (since the gradient is a unit vector)} \\ \sigma_\epsilon^2 &= \sin^4(\epsilon) * (\sin^2(\lambda) * \sigma_{xz}^2 + \cos^2(\lambda) * \sigma_{yz}^2 \\ &\quad + 2\cos(\lambda)\sin(\lambda) \langle \sigma_{xz}\sigma_{yz} \rangle) \end{aligned}$$

These uncertainties are proportional to the apriori uncertainty assigned to the geopotential heights. However, to reflect the actual uncertainties in the estimation, as measured by the residuals to the fit, the uncertainties in azimuth and elevation must be scaled by the ratio of actual RMS deviation to that expected for the apriori uncertainty, based on the following derivation.

The root mean square residual is given by

$$\text{RMS} = [(\sum \epsilon_i^2)/N]^{1/2}$$

where  $\epsilon_i$  is the residual to the fit of the estimated plane for the  $i^{\text{th}}$  point and  $N$  is the number of points used in the estimation. For estimating three parameters the goodness of fit per degree of freedom is given by

$$\chi^2/\text{dof} = \sum(\epsilon_i^2/\sigma_i^2)/(N-3)$$

This quantity should be approximately 1.0 if the uncertainties,  $\sigma_i$ , have been evaluated correctly. In addition, the same uncertainty is assumed to apply to all points. Therefore, the value of  $\sigma_s$ , the scaled apriori uncertainty in the measurement (z200), can be found as

$$\sigma_s = [(\sum \epsilon_i^2)/(N-3)]^{1/2}$$

or, in terms of the RMS,

$$\sigma_s = [N/(N-3)]^{1/2} * \text{RMS}$$

Since the weight matrix is proportional to the apriori uncertainty in the geopotential heights,  $\sigma_0$ , the uncertainty in the estimated parameters can be corrected by the ratio,  $\beta$ , of the scaled uncertainty needed to give  $\chi^2/\text{dof}$  approximately 1.0,  $\sigma_s$ , to the apriori uncertainty,  $\sigma_0$ .

For  $N = 4$

$$\beta = \sigma_s / \sigma_0 = 2 * \text{RMS} / \sigma_0$$

Then

$$\sigma_{\lambda_s} = \beta \sigma_{\lambda}$$

$$\sigma_{e_s} = \beta \sigma_e$$

$$\sigma_{z0_s} = \beta \sigma_{z0}$$

are the scaled uncertainties for azimuth, elevation, and estimated geopotential height above the site.

### 3.3. Comparison with previous gradient models

Two other versions of gradient model are in use. Both add to the azimuthally symmetric delay an asymmetric term. Chen and Herring (1997) use a continued fraction in  $\sin(e) \tan(e)$ , retaining only one term plus a constant, giving a line of sight delay contribution:

$$t(\mathbf{e}) = \frac{1}{\sin(\mathbf{e})\tan(\mathbf{e}) + c} [G_N \cos(\mathbf{a}) + G_E \sin(\mathbf{a})]$$

The  $G$  terms give the magnitude of the gradient in the north and east directions and have values on the order of 1 mm.  $\alpha$  is the azimuth, measured east from north. For the hydrostatic gradient, CH found a value of  $c = 0.0032$ . For the wet gradient  $c = 0.0010$ . However, CH used the hydrostatic value for their analysis of VLBI data.

MacMillan (1995) uses the gradient formulation of Gardner (1977):

$$t(\mathbf{e}) = m_h(\mathbf{e}) \cot(\mathbf{e}) [G_N \cos(\mathbf{a}) + G_E \sin(\mathbf{a})]$$

where  $m_h$  is the hydrostatic mapping function, MTTh, of Herring (1992). Other than the use of MTTh, MacMillan's gradient mapping function does not distinguish between hydrostatic and wet.

Since the parameterizations among the gradient formulations are different (change of elevation for IMFh, different elevation dependence between CH and MacMillan), a common elevation must be chosen for comparison. For geodetic VLBI, observations to  $5^\circ$  are reasonable, while data have been taken as low as  $3^\circ$ , so use  $5^\circ$  as the reference elevation. (CH used  $10^\circ$  for comparison with gradients determined from a NWM.) Of course the effect of the gradient should be negligible at zenith. CH found that a typical gradient parameter,  $G_N$  or  $G_E$ , is 1 mm, so this value will be used for CH and for MacMillan. The following comparison is for the hydrostatic gradient. I have used NMFh with latitude  $42^\circ$ , day-of-year 28, and a site at sea level to represent IMFh. For IMFh the elevation difference at  $5^\circ$  corresponding to 1 mm of gradient delay for CH (with  $c = 0.0032$ ) is  $0.0248^\circ$ .

A comparison of the two forms for elevations down to  $3^\circ$  is given in the following tables. In place of MTTh in MacMillan's formulation, NMFh (Niell 1996) was used.

However, as shown in Niell (1996), NMFh and MTTh agree sufficiently for this comparison. Both the hydrostatic and wet mapping functions are compared.

Table 1. Gradient delays for ZHD of 2300 mm:

elevation(°)	nmfh(ε)	nmfh(ε)- nmfh(ε+Δε) (mm)	CH (mm)	DSM (mm)
90°	1.0000	0.0002	0.0000	0.0000
30°	1.9928	3.4062	3.4261	2.7545
15°	3.8011	13.6680	13.7835	11.3188
10°	5.5546	29.2605	29.5693	25.1268
7°	7.2050	48.5724	49.0578	43.6355
5°	10.1451	92.6462	92.3776	92.3795
3°	14.6807	180.4407	168.2705	222.8192

Table 2. Difference to NMFh with tilt

elevation(°)	nmfh(ε)	nmfh(ε)- nmfh(ε+Δε) (mm)	nmfh-CH (mm)	nmfh-DSM (mm)
90°	1.0000	0.0002	0.0002	0.0002
30°	1.9928	3.4062	-0.0199	0.6517
15°	3.8011	13.6680	0.1155	2.3492
10°	5.5546	29.2605	-0.3088	4.1337
7°	7.2050	48.5724	-0.4855	4.9369
5°	10.1451	92.6462	0.2687	0.2667
3°	14.6807	180.4407	12.1702	-42.3785

Three points should be noted from Table 2:

1. For the tilted atmosphere model of the gradient (IMFh), the error at zenith is less than 1 micron of delay. It can still be considered to be normalized.
2. Line of sight delays calculated for the tilted atmosphere and for the CH form of the hydrostatic gradient can agree to better than 1 mm at all elevations down to 5°.
3. The MacMillan and CH forms of the gradient mapping function give line of sight delays that agree to better than 5 mm down to 5° for the same gradient coefficient of 1 mm. The MacMillan form should not be used for elevations below 5°.

## 4. Azimuthally symmetric wet mapping function

### 4.1. The wet parameter smfw3

The wet parameter is designated smfw3 and is given by

$$smfw3 = \frac{t(3.3^\circ)}{t(90^\circ)}$$

where  $t$  is the line of site delay at the specified elevation, given by the integral of the wet refractivity. The wet refractivity consists of two terms, one proportional to  $e_v/T$ , the other proportional to  $e_v/T^2$ , where  $e_v$  is the water vapor pressure (hPa) and  $T$  is temperature (K). I have retained only the second term because it is much larger (by a factor of approximately 80) than the  $e/T$  term.

Writing the delay in terms of the wet refractivity gives:

$$smfw3 = \frac{k_3 \int \frac{e_v(s)}{T^2(s)} ds}{k_3 \int \frac{e_v(h)}{T^2(h)} dh}$$

where  $s$  is the distance along the geometric raypath at an elevation of  $3.3^\circ$ , approximately the outgoing refracted direction for a ray that will exit the atmosphere at an angle of  $3^\circ$ . (Since the development of this algorithm, I have realized that the parameter could possibly be better defined if the refractivity at the surface were used as an input.)

(Note that the mapping function is not uniquely determined by the ratio of the delay at the lowest elevation angle to the ZWD. In this sense choosing a single parameter to represent the mapping function limits the accuracy, even though other information is used in the calculation of the three parameters of the continued fraction.)

The values of the water vapor pressure and temperature at the standard pressure levels are used as piecewise linear approximations to the integral. The heights,  $h$ , at the corresponding pressures are obtained as the geopotential heights. Each of these quantities is contained in a file that is an output of the numerical weather model. Although water vapor pressure is not output as a data type, it can be derived from specific humidity,  $q$ , as:

$$e_v = qP \frac{m_d}{m_w}$$

where  $P$  is total pressure (hPa),  $m_d$  and  $m_w$  are the mean molecular weight of dry air and water. The ratio  $m_w/m_d = 0.622$ . In DAO and ECMWF the units of specific humidity are gm/kg, so a factor of 0.001 must be applied to use  $e_v$  in hPa.

For an azimuthally symmetric atmosphere the meteorological parameters are a function of height only, so  $smfw3$  can be written as:

$$smfw3 = \frac{k_3 \int \frac{e_v(h)}{T^2(h)} \frac{ds}{dh} dh}{k_3 \int \frac{e_v(h)}{T^2(h)} dh}$$

I have retained the constant,  $k_3$ , in both the numerator and denominator to reflect the physical meaning, but they obviously cancel in the calculation.

The wet refractivity,  $n_w(h_i)$ , at the geopotential height of each pressure level is given by:

$$n_w(h_i) = 0.001 \cdot \left( \frac{k_3}{0.622} \right) \cdot \frac{q(h_i) \cdot P(h_i)}{T(h_i)^2}$$

where ( $k_3 = 377600$ ;  $\epsilon_0 = 3.3$ ). The distance  $s(h)$  along the raypath at elevation  $\epsilon_0$  from a point  $R_e$  to a height  $h$  above the surface is

$$s(h) = R_e \left\{ \left[ \left( 1 + \frac{h}{R_e} \right)^2 - \cos^2(\mathbf{e}_0) \right]^{1/2} - \sin(\mathbf{e}_0) \right\}$$

where  $R_e$  is the radius of the Earth, and  $\mathbf{e}_0$  is the elevation angle of the outgoing ray, chosen to represent the lowest elevations currently used in geodetic observations.

Using the average refractivity within a height interval  $dh$ , the numerator,  $losw$ , and denominator,  $zwd$ , of  $smfw3$  are

$$losw = 0.5 \cdot \sum_{i=2}^N [n_w(h_i) \cdot n_w(h_{i-1})] \cdot (s(h_i) - s(h_{i-1}))$$

$$zwd = 0.5 \cdot \sum_{i=2}^N [n_w(h_i) + n_w(h_{i-1})] \cdot (h_i - h_{i-1})$$

and  $smfw3$  is calculated at each grid point of the NWM as

$$smfw3 = \frac{losw}{zwd}$$

#### 4.2. The wet mapping function (revised 2003/05/05)

After modification of the calculation of  $smfw3$  to use the raypath length  $s(h)$  given above, the form of the wet mapping function was revised to account for the dependence on height of the site above sea level. This is included in the  $a$  coefficient of the continued fraction. In addition, the dependence on  $smfw3$  is referenced to a value of 15.5, which is approximately the median value among the radiosonde sites used for determination of the coefficients.

The three coefficients of the wet mapping function continued fraction are given by:

$$\begin{aligned} a0 &= 6.8827e-004; \\ a_{ht} &= -1.6580e-007; \\ dads &= -2.0795e-004; \\ b0 &= 1.3503e-003; \\ dbds &= 1.8882e-004; \\ c0 &= 3.9647e-002; \\ dcds &= 4.8581e-003; \\ smfw0 &= 15.5; \end{aligned}$$

$$\begin{aligned} a &= (smfw3 - smfw0) \cdot dads + a0 + a_{ht} \cdot ht; \\ b &= (smfw3 - smfw0) \cdot dbds + b0; \\ c &= (smfw3 - smfw0) \cdot dcds + c0; \end{aligned}$$

The derivative of the mapping function with respect to elevation,  $dm/d\mathbf{e}$ , is needed for calculating the partial derivative with respect to time.

$$num = 1 + \frac{a}{1 + \frac{b}{1+c}}$$

$$denom = \sin(\mathbf{e}) + \frac{a}{\sin(\mathbf{e}) + \frac{b}{\sin(\mathbf{e}) + c}}$$

$$t1 = \sin(\mathbf{e}) + c$$

$$t2 = \sin(\mathbf{e}) + \left( \frac{b}{t1} \right)$$

$$\begin{aligned} \frac{dm}{d\mathbf{e}} &= -\frac{num}{denom^2} \cdot \left[ \cos(\mathbf{e}) - \frac{a \cdot \cos(\mathbf{e})}{(t2)^2} \cdot \left( 1 - \frac{b}{(t1)^2} \right) \right] \\ &= -\frac{num}{denom^2} \cdot \left[ \cos(\mathbf{e}) - \frac{a \cdot \cos(\mathbf{e})}{(\sin(\mathbf{e}) + (b/(\sin(\mathbf{e}) + c)))^2} \cdot \left( 1 - \frac{b}{(\sin(\mathbf{e}) + c)^2} \right) \right] \end{aligned}$$

## 5. Wet gradient mapping function

After removing the apriori hydrostatic gradient mapping function, the most appropriate wet gradient mapping function should be used to estimate the remaining atmosphere asymmetry. The recommended gradient mapping function is that of CH using the parameter  $c = 0.0010$ . This value was found by CH to best agree with the characteristics determined from raytracing of the data from a NWM. However, the grid spacing for the NWM corresponds to spatial separations of approximately 200 km. Water vapor is known to have significant variation in density over much smaller scales, so the analysis of CH should be repeated with much higher resolution weather models to improve the wet gradient mapping function.

## 6. References

- Gardner, C. S., Correction of laser tracking data for the effects of horizontal refractivity gradients, *Appl. Opt.*, **16**, 2427-2432, 1977.
- Herring, T. A., Modelling atmospheric delays in the analysis of space geodetic data, in Symposium on Refraction of Transatmospheric Signals in Geodesy, J. C. DeMunk and T. A. Spoelstra, eds., Netherlands Geodetic Commission Series No. 36, 157-164, 1992.
- MacMillan, D. S., Atmospheric gradients from very long baseline interferometry observations, *Geophys. Res. Lett.*, **22**, 1041-1044, 1995.
- Niell, A. E., Preliminary evaluation of atmospheric mapping functions based on numerical weather models, *Phys. Chem. Earth*, **26**, 475-480, 2001.
- Schubert, S. D., J. Pjaendtner, and R. Rood, An assimilated data set for Earth science applications, *B. A. M. S.*, **74**, 2331-2342, 1993.