EDGES MEMO #076

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Subject: Noise wave analysis using reflection coefficients referenced to 50 ohms

The noise wave representation of the energy transfer from the antenna to the LNA has some advantages over a pure circuit model. For example consider transfer of sky noise from an antenna with impedance, Z_{ant}, to an LNA with impedance Z_{lna}. The reflection coefficient, Γ, is

$$\Gamma = \left(Z_{ant} - Z_{\mathrm{lna}}^{*}\right) \Big/ \left(Z_{ant} + Z_{\mathrm{lna}}\right) \tag{1}$$

And the noise power from sky temperature, T_{sky}, into the LNA is

$$T_{sky}\left(1 - \left|\Gamma\right|^{2}\right) \tag{2}$$

However it is more convenient to measure the reflection coefficients in a 50 ohm system in which case the power to the noise power to the LNA is

$$T_{sky}\left(1 - \left|\Gamma_{a}\right|^{2}\right)\left|F\right|^{2} \tag{3}$$

Where

$$F = \left(1 - \left|\Gamma_{\ell}\right|^{2}\right)^{1/2}\left[\sum_{K=0}^{\infty}\left(\Gamma_{a}\Gamma_{\ell}L\right)^{K}\right] = \left(1 - \left|\Gamma_{\ell}\right|^{2}\right)^{1/2} \Big/ \left(1 - \left(\Gamma_{a}\Gamma_{\ell}L\right)\right) \tag{4}$$

and where Γ_{a} and Γ_{ℓ} are the reflection coefficients of the antenna and LNA respectively with respect to 50 ohms. The complex factor F is the sum of noise voltage waves back and forth between the antenna and LNA via a 50Ω attenuator or cable with one way power loss factor L. The polylogarithmic series converges when $\left|\Gamma_{a}\Gamma_{\ell}L\right| < 1$ and has an exact solution.

We can add the contributions of the noise from the attenuator and LNA to equation (3) as follows:

$$\begin{aligned} &T_{sky}\left(1 - \left|\Gamma_{a}\right|^{2}\right)L\left|F\right|^{2} + T_{amb}\left[\left(1 - L\right)L\left|\Gamma_{a}\right|^{2} + \left(1 - L\right)\right]\left|F\right|^{2} + T_{LNAU}\left|\Gamma_{a}\right|^{2}L^{2}\left|F\right|^{2} \\ &+ T_{LNAC}\left|\Gamma_{a}\right|\left|L\right|\left|F\right|\cos\phi + T_{LNA} \end{aligned} \tag{5}$$

Where T_{amb} is the temperature of the attenuator. The noise from the LNA is in 3-parts are as follows:

T_{LNAU} = noise towards antenna which is uncorrelated with other sources of noise

T_{LNAC} = noise towards antenna which is correlated with other noise from LNA

T_{LNA} = noise contribution from LNA when connected to a 50 Ω load.

The correlated noise from the LNA depends on the difference between the phase of the reflection, ϕ_r, and the correlation phase, ϕ_c, where

$$\phi_r = \text{phase of } \Gamma_a F \tag{6}$$

Solving for the 4 LNA noise parameters, T_{LNAU}, T_{LNAC}, T_{LNA} and ϕ_c

1] Connect matched noise source to convert power to temperature
2] Connect matched load

$$T = T_{amb} + T_{LNA} \tag{7}$$

3] Connect open cable with measured Loss L so $\Gamma_a = e^{-2i\omega\tau}$ where ω is the angular frequency and τ the cable's delay.

$$\begin{aligned} &T = T_{amb}\left[1-L^2\right]|F|^2 + T_{LNAU}L^2|F|^2 \\ &T_{LNAC}L|F|\cos\left(\text{phase of } \Gamma_a F - \phi_c\right) + T_{LNA} \end{aligned} \tag{8}$$

$T_{amb}\left[1-L^2\right]|F|^2$, is known from the measurements the cable and the LNA input impedances, along with T_{LNA} can be subtracted from equation (8). The remaining terms can be found by fitting functions of $L^2|F|^2$, $L|F|\cos\left(\text{phase of } \Gamma_a F\right)$ and $\mathrm{L}|F|\sin\left(\text{phase of } \Gamma_a F\right)$ to determine T_{LNAU}, T_{LNAC} and ϕ_c. For EDGES the following are typical results at 100 MHz:

T_{LNA} = 43 K

T_{LNAU} = 40 K

T_{LNAC} = 37 K

ϕ_c = 108 degrees

In summary measurements made of antenna and LNA input impedance along with EDGES measurements of an open cable to derive the LNA noise waves can provide a complete calibration of EDGES.