To: EDGES Group
From: Alan E.E. Rogers
Subject: Low loss torodial air core balun

1] Introduction
A balun needs to have the lowest possible resistive loss. The “compensated choke” balun described in memos 43 and 72 has about 0.25 dB loss at 60 MHz but the loss in the ferrite is mainly resistive rather than reflective as would be the case if the ferrite core impedance was reactive.

A better balun might be made by relying only on low loss reactive components which means avoiding the use of ferrite or iron core materials. Loss factor of the balun used in making a correction for the antenna temperature approaches unity as the choke inductance becomes entirely reactive. This condition can be achieved by using an air core toroid for which the loss is very small and is dominated by the skin effect losses of the coaxial cable used in the winding.

2] Toroidal case inductance
The inductance of a toroidal core is approximately given by:

\[ L = \frac{\mu N^2 \pi r^2}{2\pi R} + \frac{2\pi r N Z}{c} \]

where
- \( N \) = number of turns
- \( \mu = 4 \times 10^{-7} \) for air
- \( r \) = radius of winding
- \( c = 3 \times 10^8 \)
- \( Z = \) impedance of open wire ≈ 200 \( \Omega \)

For \( N = 16, r = 1 \) cm, \( R = 2 \) cm, \( L = \approx 1.5 \mu H \)

3] Skin effect losses
An 100 MHz the skin depth in copper is 6.6 \( \mu \)m and decreases with the square root of frequency. The copper loss are made up of the skin effect in the center conductor of the first \( N/2=8 \) turns which amounts to about 1\( \Omega \) and the skin effect all \( N \) turns of the outer conductor which amounts to about 0.5\( \Omega \).
4] Equivalent circuit
The equivalent circuit is the same as for the “compensated choice” balun where the total impedance across the balanced output is the inductance of 1.5 \( \mu \)H in series with a resistance of 0.4 \( \Omega \). The center conductor loss results in the cable loss of about 0.1 dB at 100 MHz.

5] Loss factor of toroidal balun
If we assume that the antenna impedance is measured through the balun the loss in coax (which is significantly higher in the toroidal balun than in the ferrite balun) cannot be accurately taken into account as an additional attenuator. The antenna temperature at the balun output is

\[
T_{ant} = \left( T_{sky} \cdot L \left( 1 - \left| \Gamma_a \right|^2 \right) + T_{amb} \left( 1 - L \right) + T_{amb} \left( 1 - L \right) L \left| \Gamma_a \right|^2 \right) c
\]

where \( c = 1/(1 - \Gamma_a \Gamma f L) \)
\( \Gamma_a \) = antenna reflection at input to coax in balun
\( \Gamma f \) = LNA reflection coefficient
\( L \) = loss factor of coax in the balun

\[
T_{ant} = \left( T_{sky} \cdot L \left( 1 - \frac{T^2}{L^2} \right) + T_{amb} \left( 1 - L \right) + T_{amb} \left( 1 - L \right) \frac{T^2}{L} \right) c
\]

since \( \Gamma = \) antenna reflective measured at the input of balun
= \( \Gamma_a L \)

From the above we find the loss factor for the cable in the balun\(^1\)

\[
L_f = L \left( 1 - \frac{T^2}{L^2} \right) / \left( 1 - \Gamma^2 \right)
\]

So that

\[
T_u = T_{sky} L_f + \left( 1 - L_f \right) T_{amb}
\]

Where \( T_u \) is the calibrated antenna temperature “uncorrected” for the balun loss. \( L \) can be measured with a VNA measurement of the balun with balanced output shorted. The loss in the “core” of the balun can be measured from the difference of the reflection magnitude with the balanced output open and the balanced output shorted. Since the “core” loss is very small great care is needed in the VNA calibration. The parallel impedance, \( Z_f \), can be estimated from this measurement and used to calculate the “core” loss factor.

\[
L = \text{Re}(Z_a) \left| Z_f \right|^2 / \left[ \text{Re}(Z_a) \left| Z_f \right|^2 + \text{Re}(Z_f) \left| Z_a \right|^2 \right]
\]

\(^1\) This result can also be found in Microwave Engineering by David M. Pozar.
In summary the antenna temperature is first obtained from measurements of the spectrum calibrated in Kelvin from the 3-position switch. This is then corrected for the noise waves and the antenna to LNA mismatch based on impedance measurements of the LNA and antenna at the reference plane defined at the junction of the 3-position switch input and balun output. Then corrections are made for the loss of the cable in the balun, the losses in the impedance the balun, antenna resistive loss and ground loss.

6] Comparison with ferrite and air core
The table below shows the effect of antenna impedance on the balun loss factor. The ferrite balun has more loss and the loss is more sensitive to the antenna impedance.

<table>
<thead>
<tr>
<th>Freq. (MHz)</th>
<th>Ant impedance</th>
<th>Ferrite (K)</th>
<th>Air core toroid (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>50</td>
<td>96.38</td>
<td>31.43</td>
</tr>
<tr>
<td>70</td>
<td>50 + 50 j</td>
<td>193.66</td>
<td>47.32</td>
</tr>
<tr>
<td>70</td>
<td>50 – 50j</td>
<td>194.48</td>
<td>47.32</td>
</tr>
<tr>
<td>70</td>
<td>100</td>
<td>195.29</td>
<td>39.41</td>
</tr>
<tr>
<td>70</td>
<td>25</td>
<td>49.03</td>
<td>39.31</td>
</tr>
</tbody>
</table>

7] Initial tests
An initial test was made using RG174 cable. Measurements were made which showed that the loss was equal to the cable loss for 8 turns plus 2 cm for a total of 52 cm of RG-174 for a loss of 0.2 dB at 100 MHz. The 2-way delay was 6 ns and started increasing at 50 MHz. Below 50 MHz the reactive impedance of the 1.5 µH becomes significant. The common mode rejection was measured and found to exceed 40 dB. The next step was to construct a balun using low loss handformable 0.085 semi-rigid cable to replace the RG-174. This balun is shown in Figure 1. This balun had a measured Loss of 0.15 dB at 150 MHz. Unfortunately it was found that the air core has some subtle resonances due to modes not initially considered. For example, when the length of the coax in the core is equal to a quarter wavelength, which for the core made of 0.085 semi-rigid occurred at 70 MHz, there is a subtle resonance which was not seen initially. The result is that the frequency range of the air core toroid is more limited than that of the ferrite balun in which the length of coax is extremely short. The toroidal balun could still be used over a frequency range which includes the 70 MHz resonance but there is another resonance at 124 MHz at which the loss increases to 0.6 dB compared with 0.4 dB for the balun with 2 ferrite cores on each “leg.” These resonances are most evident when there is an imbalance between common and differential mode loads. While the air core toroidal balun could still be better than a ferrite core balun it is clear that an accurate circuit and noise model is more complex.
Figure 1. Air core toroid balun using 0.085 semi-rigid cable. The balanced input, on the left, uses a short length of 50 ohm coax as there is no standard 50 ohm balanced cable and connector.