EDGES MEMO #090

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To: EDGES Group
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Subject: Additional calibration steps for improving VNA accuracy

The standard short, open, load (SOL) method is used to calibrate the VNA. In this method the directivity, e_{00}, mismatch, e_{11}, and tracking, e_{10} are derived by solving

$$\Gamma_m = e_{00} + e_{10}\,\Gamma_t \big/ \left(1 - e_{11}\Gamma_t\right)$$

Where Γ_t is the "true" reflectivity and Γ_m is the measured reflectivity.

The solutions are

$$e_{00} = \Gamma_{load}$$

$$e_{11} = \left(2e_{00} - \left(\Gamma_{open} + \Gamma_{short}\right)\right) \Big/ \left(\Gamma_{short} - \Gamma_{open}\right)$$

$$e_{10} = \left(\Gamma_{open} - e_{00}\right)\left(1 - e_{11}\right)$$

and $\Gamma_{cor} = \left(\Gamma_{obs} - e_{00}\right) \big/ \left(e_{10} - e_{00}\,e_{11} + \Gamma_{obs} e_{11}\right)$

where Γ_{open}, Γ_{short} and Γ_{load} are the measured reflectivities and Γ_{cor} is the corrected reflection coefficient obtained from the observed reflection coefficient.

In the low frequency range of interest to EDGES the SOL calibration is mainly limited by errors in the calibration load which can be eliminated by including an accurate measurement of the DC resistance of the load.

Since the VNA internal software makes the SOL calibration corrections a practical procedure is to follow with these "post calibration" steps:

1] Perform the standard SOL calibration

2] Measure the DC resistance, R_L, of the load (or another load). For 2-lead measurement be sure to subtract the lead resistance.

3] Measure the calibrated Γ_{cl} of the load.

4] Derive and apply $e'_{00}, e'_{10}, e'_{11}$ to further correct the calibrated VNA data

$$e'_{00} = \Gamma_{cl} - \left(R_L - 50\right) \big/ \left(R_L + 50\right)$$

$$e'_{11} = -e'_{00}$$

$$e'_{10} = (1 - e'_{00})(1 - e'_{11})$$

Use $e'_{00}, e'_{11}, e'_{10}$ to correct the SOL calibrated deflection coefficients:

$$\Gamma_{cal} = (\Gamma_{VNA} - e'_{00}) / (e'_{10} - e'_{00}\, e'_{11} + \Gamma_{VNA}\, e'_{11})$$

$$= (\Gamma_{VNA} - e'_{00}) / (1 - e'_{00}\, \Gamma_{VNA})$$

Where Γ_{VNA} is the reflection coefficient from the SOL calibrated VNA. Γ_{cal} is the result of applying the additional correction made "offline" to improve the accuracy of the VNA. Since a typical calibration load differs from 50 ohms by about 0.5 ohms the SOL calibration is only accurate to about 0.1 dB for a reflection coefficient near -15 dB. When high quality calibration shorts, opens and loads are purchased the offset delays and other parameters associated with the shorts and opens accompany them but the loads are assumed to be perfect and no individual data is supplied. For the tests of this method described in the next section we call the post SOL calibrated results enhanced SOL or ESOL for short.

In order to test the accuracy of the method terminations with precisely known reflection coefficient are needed. In the low frequency range attenuators can be used for this purpose. The reflection magnitude can be precisely determined by measurement of the DC resistance of each port R_a, R_b and the resistance between ports R_{ab}. From these the equivalent "T" network resistances are

$$r_a = (R_a - R_b + R_{ab})/2$$

$$r_b = R_{ab} - r_a$$

$$r_c = R_a - r_a$$

Where r_c is the common resistor to ground. For example, the impedance of part A with part B open is

$$Z_a = r_a + r_c$$

And the reflection coefficient is

$$(Z_a - 50)/(Z_a + 50)$$

While the phase of the reflection coefficient can be estimated from the SOL calibrated measurement the magnitude is more accurately known from the DC resistance measurements than from the VNA SOL calibrated measurement. In this case we can only check the accuracy of the ESOL calibrated results in magnitude but for EDGES the error in the magnitude of the reflection coefficients is the largest source of error.

For a test of the ESOL method I calibrated on Agilent N5222A using a Maury 8050B SOL kit and then measured the reflection coefficient on port A of a 6 dB attenuator with R_a=85.9 Ω, R_b = 85.8 Ω, R_{ab} = 33.0 Ω. The DC resistance of the Maury load was 49.4 Ω. The following results were obtained for the attenuator reflection magnitude.

Frequency (MHz)	SOL (dB)	ESOL (dB)	Calculated
50	-11.387	-11.557	-11.563
100	-11.392	-11.561	-11.563
150	-11.396	-11.565	-11.563
200	-11.403	-11.568	-11.563

This simple test illustrates the potential for a substantial improvement in reflection coefficient magnitude by simply correcting the error in the calibration load based on an accurate measurement of its DC resistance.