EDGES MEMO #125

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To: EDGES Group

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Subject: The dependence the Roberts balun loss on antenna reflection phase

At the relatively low frequencies of EDGES coaxial transmission lines have a complex impedance. For example the coaxial balun tube which has an impedance close to 50 ohm has an imaginary value which ranges from -0.1 to -0.05 ohms from 50 to 200 MHz. This imaginary component has the effect of making the loss of the transmission line depend on the phase of the reflection coefficient of the source at its input. The same effect is present in the loss of VNA calibration opens and shorts. At low frequencies the loss of the short is much larger than the loss of the open. The effect is also demonstrated for an open and shorted cable in memo 84.

The noise contribution from the cable can be calculated from the difference between the total noise when the cable is connected to an ambient load and the noise when a noiseless cable is connected to an ambient load.

The total noise is given by

$$\text{noise1} = \operatorname{Re}\left(iZ\ell i^*\right)$$

where $i = 2\left(T_{amb} \operatorname{Re} Zss\right)^{1/2} / \left(Zss + Z\ell\right)$

$Zss = Zcab\left(1+T\right)/\left(1-T\right)$

$T = \left[\left(Zs - Zcab\right)/\left(Zs + Zcab\right)\right]e^{-2\gamma\tau c}$

Zcab is the complex cable impedance

$\gamma$ is the complex propagation constant

$\tau$ is the one-way delay in the cable

c is the propagation velocity

Zs is the impedance of the antenna

Zss is the impedance of the antenna seen through the cable

$Z\ell$ is the LNA impedance

The noise contribution from the antenna connected to a cable at zero kelvin can be calculated from the transmission of the antenna noise through the cable. In this case

$$\text{noise2} = \mathrm{Re}\left(vv^*/Z\ell^*\right)$$

where $V^+ = \tfrac{1}{2}\left(Zcab + Z\ell\ell\right)/\left(Zs + Z\ell\ell\right)$

$$V^- = \tfrac{1}{2}\left(Z\ell\ell - Zcab\right)/\left(Zs + Z\ell\ell\right)$$

$$v = 2\left(V^+ e^{-\gamma\tau c} + V^- e^{+\gamma\tau c}\right)\left(T_{amb}\,\mathrm{Re}\,Zs\right)^{1/2}$$

where $Z\ell\ell$ is the LNA impedance seen through the cable. The transformation of the antenna impedance, Zs, to that seen looking through the cable, Zss, is given by

$$T = \left[\left(Zs - Zcab\right)/\left(Zs + Zcab\right)\right]e^{-2\gamma\tau c}$$

$$Zss = Zcab\left(1+T\right)/\left(1-T\right)$$

A similar expression converts $Z\ell$ to $Z\ell\ell$. The reverse transformation requires changing the sign of the exponent to $2\gamma\tau c$. Using the dimensions of the tube and center conductor of the Roberts balun the complex impedance and propagation factors are calculated. The resistance, R, is calculated assuming copper. See memo #86 for formulae.

Figure 1 shows the thermal noise from the 0.5m long balun cable as a function of the phase of an antenna reflection coefficient of 0.15 at 100 (lower curve) and 200 MHz (upper curve).

The cable loss factor, L, is given by

$$L = \left(noise2/T_{amb}\right)/corr$$

where $corr = \left(1-\left|\Gamma_a\right|^2\right)\left(1-\left|\Gamma_\ell\right|^2\right)/\left|1-\Gamma_a\Gamma_\ell\right|^2$

$$\Gamma_a = \left(Zss - 50\right)/\left(Zss + 50\right)$$

$$\Gamma_\ell = \left(Z\ell - 50\right)/\left(Z\ell + 50\right)$$

The ambient temperature cancels so that the loss can be calculated with $T_{amb}$ = 1 in which case

$$L = \left(\mathrm{Re}\,vv^*/Z\ell^*\right)/corr$$

$$v = 2\left(V^+ e^{-\gamma\tau c} + V^- e^{+\gamma\tau c}\right)\left(\mathrm{Re}\,Zs\right)^{1/2}$$

The corrected sky noise (assuming a lossless antenna is given by

$$T_{sky} = \left(T_{measured} - T_{amb}\left(1-L\right)\right)/L$$

The measured temperature is the fully calibrated temperature which has been corrected for the reflection coefficient of the antenna and LNA as seen from a reference plane at the end of the cable at the input to the LNA. This is the same reference plane at which the hot/cold load and LNA noise waves were measured.

While the expressions which makeup the L factor depend on $Z\ell$ the final result is independent of $Z\ell$ and for 50 ohm cable the factor becomes

$$L = e^{-2\,\mathrm{Re}(\gamma)\tau c}\left(1-\left|\Gamma_a\right|^2 \Big/ e^{-4\,\mathrm{Re}(\gamma)\tau c}\right)\Big/\left(1-\left|\Gamma_a\right|^2\right)$$

which shows the dependence on source match and independence on source phase. Since L is the independent of $Z\ell$ the formula for L is simplified by making $Z\ell$=Zcab in which case $Z\ell\ell$ is also equal to Zcab.

Figure 2 shows the estimated thermal noise from the cable given by (noise1-noise2) as a function of frequency for coaxial section of the EDGES-2 antenna using the measured S11. These results were checked using direct integration of the telegrapher's equations to add up the noise contributions for each resistor in the circuit model elements.

It has also been shown, using simulation, that any cable added on the LNA side of the reference plane is transparent has no effect on the final calibrated result. As the effects of the source phase dependent loss thermal noise becomes part of the LNA impedance and noise waves which are removed through calibration.

Comments:

While the temperature contribution of the balun and any additional cable needed to connect to the LNA is small it was not appreciated how much structure is present. This structure would be reduced if the antenna reflection is reduced. A 6 dB drop in antenna reflection coefficient reduces the phase dependence by a factor of 2. While the phase dependence increases with the inverse square root of frequency it is reduced by the square root of 2 for a low band antenna whose balun tube diameter is double that of the high band antenna.

Since the frequency structure depends on the phase of the antenna prior to the balun it doesn't help to use a long cable, as proposed in earlier memos, to improve EoR detectability by averaging over frequency. The effect could be reduced by increasing the diameter of the balun tubes. A factor of 2 increase would reduce the effect by a factor of 2.

While using VNA measurements of the balun open and shorted can be used to check the cable model that actual correction which needs to be applied to the data has to be modeled. In principle this should be straight forward and doesn't require very accurate knowledge of the antenna S11 phase since the sensitivity is less than about 1 mK/deg for the high band and under 5 mK/deg for the low band. The model is not very sensitive to the conductivity. For example, the difference between cooper and gold is only 20% and the variation for different types of copper should effect the loss by less than 2%.
An alternate and simpler formulation of the loss is given by Pozar in Microwave Engineering page 82-83, who considers the equivalent problem of cable loss from a transmitter. Augmenting Pozar's formulation for complex impedance:

$$T = \left[(Zss - Zcab)/(Zss + Zcab)\right]e^{2\gamma\tau c}$$

$$Vout = (1+T)$$

$$Iout = (1-T)/Zcab$$

$$Vin = \left( e^{\gamma\tau c} + Te^{-\gamma\tau c} \right)$$

$$Iin = \left( e^{\gamma\tau c} - Te^{-\gamma\tau c} \right) \Big/ Zcab$$

$$L = \mathrm{Re}\left( Vout\ Iout^{*} \right) \Big/ \mathrm{Re}\left( Vin\ Iin^{*} \right)$$

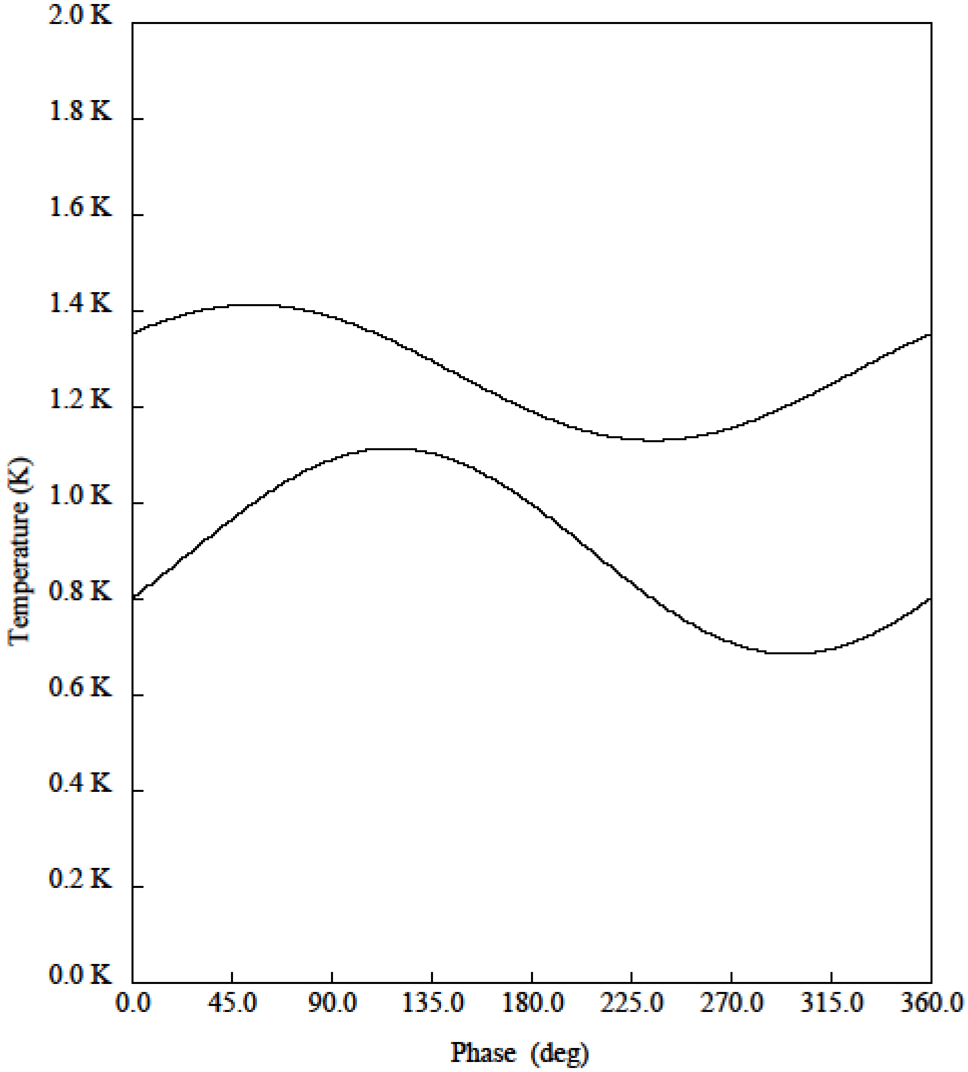


Figure 1. Temperature contribution from the cable vs phase of antenna S11 phase. The upper and lower curves are for 200 MHz and 100 MHz respectively.

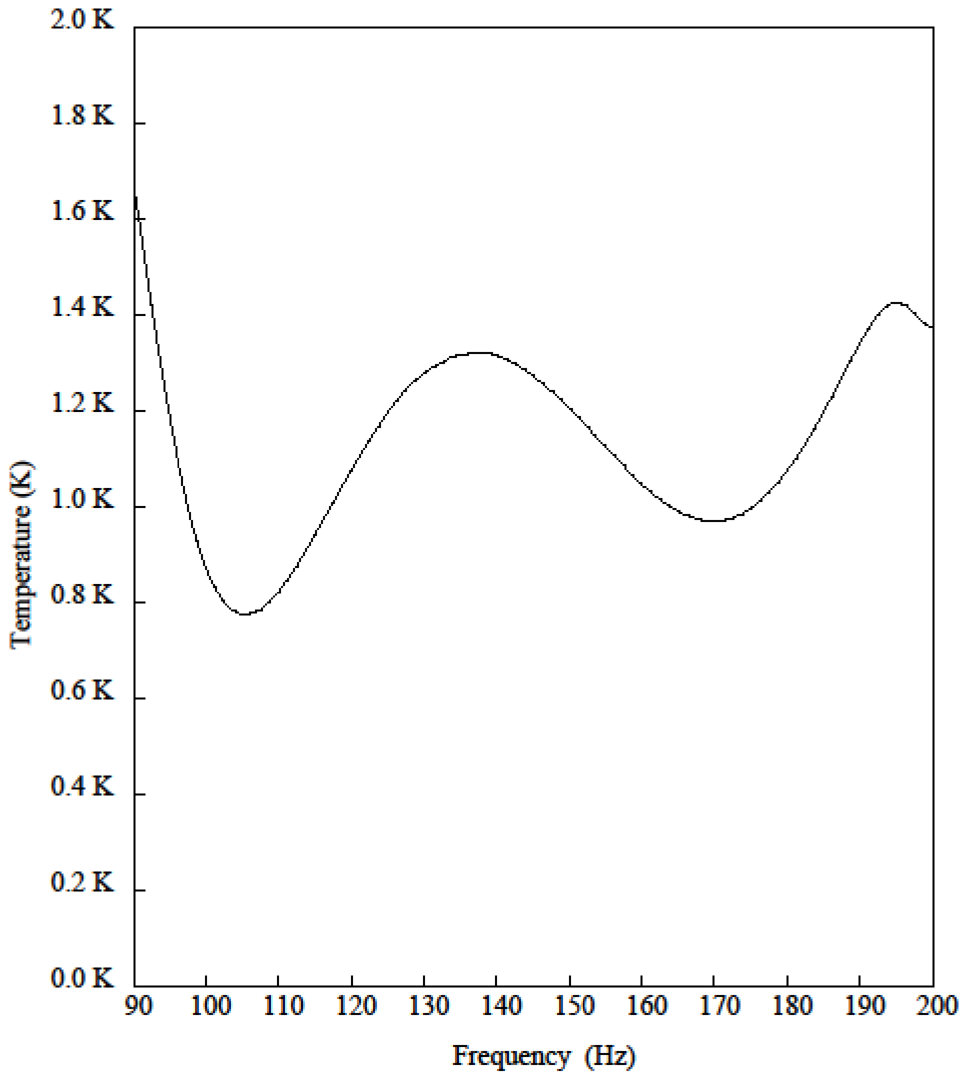


Figure 2. Estimate of temperature contribution from balun loss vs frequency.