1 Introduction

EDGES Memos 79 and 143 discuss the effect of absorption and emission for EDGES measurements, and show how the difference between two spectra can be used to estimate electron temperature and change in opacity. The equations assume one value of temperature and opacity for the ionosphere. A more accurate model would need to take into account the altitude distribution of absorption and electron temperature. This is needed, as approximately the same amount of absorption happens in the F- and D-region during normal quiet ionospheric conditions. These ionospheric regions have significantly different electron temperatures ($\approx$200 K in the D-region, and $\approx$1500 K in the F-region).

If total absorption is small, it should be possible to ignore the gradual change of the sky noise spectrum as it travels through the ionosphere and is modified through absorption and emission. The spectrum can then be expressed as

$$T(f) = T_B f^{-s} - T_B f^{-s} f^{-2} \sum_r L_r + f^{-2} \sum_r L_r T_e(r),$$  

where $T_B$ is the sky noise temperature, $-s$ is the spectral index of the sky noise, the term $f^{-2}$ arises from electromagnetic wave propagation in collisional plasma (equation 2, assuming $\nu(r) \ll \omega$), $L_r = 1 - 10^{-A_r/10.0}$ is the absorption coefficient for a narrow range interval at range $r$, and $T_e(r)$ is electron temperature at range $r$. The sum is formed across the range of all ranges that the ray propagates through, and the ionosphere is assumed to be locally homogeneous at each altitude.
The absorption coefficient $A_r$ is obtained using \[2\]

$$A_r = 4.6 \cdot 10^{-5} \int_{R_r} \frac{N_e(s)\nu(s)}{\nu(s)^2 + \omega^2} ds,$$

with the integral evaluated over a narrow range interval at range $r$. This has units of dB. The collision rate $\nu(r) = \nu_{ei}(r) + \nu_{ni}(r)$ is the sum of electron-neutral and electron-ion collisions. The first is important in the D- and E-regions of the ionosphere, and the latter is important in the F-region \[3, 1\].

If we examine the difference of two spectra $\Delta T(f) = T(f) - T'(f)$ at the same time of day, we can arrive with the expression

$$\Delta T(f) = -T_B f^{-s} f^{-2} \sum_r (L_r - L'_r) + f^{-2} \sum_r L_r T_e(r) - f^{-2} \sum_r L'_r T'_e(r).$$

Here the primed variables correspond to changed quantities in spectrum $T'(f)$.

If the temperature on two different days is assumed to be equal $T_e(r) = T'_e(r)$, then the equation can be simplified further to

$$\Delta T(f) = (-T_B f^{-s} + \tilde{T}_e) f^{-2} \Delta \tau,$$

where $\Delta \tau = \sum_r L_r - L'_r$ and

$$\tilde{T}_e = \frac{1}{\sum_r L_r - L'_r} \sum_r T_e(r)(L_r - L'_r).$$

Here $\tilde{T}_e$ corresponds to the weighted average of electron temperature over all altitudes, with the difference in opacity at each altitude $L_r - L'_r$ as the weight. This is interesting, as equation 4 is essentially the same as the equation in memo 143. This tells us that the interpretation for $T_e$ in that equation is that it is a weighted average of electron temperature weighted by the changes in ionospheric opacity, assuming that the electron temperature doesn’t change very much from day to day. In general, this is a good approximation, as the electron temperature will typically vary much less than electron density.

References
