1] Radiative transfer

For a single uniform gas cloud the antenna temperature is given by

\[ T_A = T_s e^{-\tau} + T_{\text{spin}} \frac{\Omega_c}{\Omega_B} \left(1 - e^{-\tau} \right) \quad \text{for } \Omega_c \leq \Omega_B \]

\[ = T_s e^{-\tau} + T_{\text{spin}} \left(1 - e^{-\tau} \right) \quad \text{for } \Omega_c > \Omega_B \]

where

- \( T_A \) = antenna temperature
- \( T_s \) = antenna temperature due to a small background source
- \( T_{\text{spin}} \) = spin or excitation temperature of the line
- \( \tau \) = opacity of the line
- \( \Omega_c \) = solid angle of cloud
- \( \Omega_B \) = solid angle of the antenna beam

In the general case of many clouds all with small opacity

\[ T_A = \sum K T_{sk} - \sum K T_{sk} \sum j \tau_j I_{kj} + \sum j T_{\text{spin}} \left( \frac{\Omega_j}{\Omega_B} \right) \]

where

- \( I_{kj} = 1 \) if the \( k \)th source is behind the \( j \)th cloud
- \( I_{kj} = 0 \) otherwise

In the case of only the uniform background

\[ T_A = T_b - T_b \sum j \tau_j \left( \frac{\Omega_j}{\Omega_B} \right) + \sum j T_{\text{spin}} \tau_j \left( \frac{\Omega_j}{\Omega_B} \right) \]

\[ = T_b + \left( \bar{T}_{\text{spin}} - T_b \right) \tau \]
where \( T_b \) = background brightness temperature
\( \bar{T}_{\text{spin}} \) = “average” spin temperature
\( \bar{\tau} \) = “average” opacity

In the case of a dominant background source

\[ T_A = T_b - T_b \bar{\tau} \]

where \( \bar{\tau} \) is the total opacity of all clouds along the line to the source.

2] Galactic center

If each station of 5×5 crossed-dipoles is pointed at the Galactic center the beam convolved with the sky brightness predicts an antenna temperature of about 500 K. In this case the radiative transfer gives

\[ T_A = T_b - \left(T_b - \bar{T}_{\text{spin}}\right) \bar{\tau} \]

where \( \bar{\tau} \) is now an average of the opacity weighted by the relative contributions of each source in the 12 degree antenna beam.

If SgrA* were dominant

\[ \bar{\tau} = 0.28 \left(\frac{n_D}{n_H}\right) \tau_H \]

where \( \tau_H \) is the opacity of the 21 cm hydrogen line in the line to SgrA*

and \( \left(\frac{n_D}{n_H}\right) \) is the relative abundance of deuterium atoms

The factor 0.28 is appropriate for the case that line widths are dominated by bulk motions and thermal broadening is negligible. (see Anantharamaiah and Radhakrishan 1979).

The H1 absorption feature at 0 Km/s in the direction of SgrA* is quite widespread and has an opacity of about 3.5 but a weighted average of the opacity over the Galactic center region probably no more than 2. [Several maps exist of the H1 absorption; Liszt et at, Astron. Astrophys. 126,341-351 (1983); Mebold et al., Astron. Astrophys., 115, 223-241 (1982)]

If we take the value of \( \left(\frac{n_D}{n_H}\right) \) to be \( 1.5 \times 10^{-5} \) the peak value of \( \Delta T_A / T_A \) when might expected is

\[ 0.28 \times 1.5 \times 10^{-5} \times 500 - 130 \times 2 / 500 = 6 \times 10^{-6} \]
3] Galactic anticenter

If each station is pointed toward the Galactic anticenter the beam convolved with the sky brightness predicts and antenna temperature of about 70 K. There is some uncertainty in this number as a result of some inconsistencies between surveys [see Reich and Reich, Astron J. Suppl. 74, 7-23, 1988]. In my opinion the number could be between 50 and 70 K. The radiative transfer gives

\[ T_A = T_b + (\bar{T}_{\text{spin}} - T_b) \bar{\tau} \]

where

\[ \bar{\tau} = 0.28 \frac{n_D}{n_H} \tau_H \]

where \( \tau_H \) is the weighted opacity of the H1 line of a region with the same beamwidth as the D1 array.

Dickey et al (Ap.J. Suppl. 53, 591-621, 1983) have measured the H1 absorption in the spectrum of many sources near the galactic plane and find that the opacity at 0 km/s to be about 2 in the region within a few degrees of the anticenter. The position for which the velocity crowding appears to be centered is around \( \ell = 183^\circ \) and previous observations of the D1 line in the directions of the anticenter are centered on \( \ell = 183^\circ \) rather than \( \ell = 180^\circ \). The emission in the anticenter peaks at around 130 K so that one might assume \( \bar{T}_{\text{spin}} = 130 K \). In this case we might expect the peak value of \( \Delta T_A / T_A \) to be

\[ 0.28 \times 1.5 \times 10^{-5} \times (130-70) \times 2 / 70 = 7 \times 10^{-6} \]

and if we assume an amplifier contribution of 15 K then

\[ \Delta T_A / T_{\text{sys}} \approx 7 \times 10^{-6} \times 60 / 75 = 6 \times 10^{-6} \]

making the estimate of the detectability about the same for the Galactic center and anticenter. We have assumed that all the continuum is behind the clouds. If some of the half sources are in the foreground

\[ \Delta T_A / T_A \approx 0.28 \times 1.5 \times 10^{-5} \times (130 - 35) \times 2 / 70 = 1.1 \times 10^{-5} \]