Calibration of Active Antenna Arrays Using A Sky Brightness Model

Alan E.E. Rogers, Preethi Pratap, and Eric Kratzenberg

MIT Haystack Observatory, Westford, MA USA

Marcos A. Diaz

Boston University, Boston, MA USA

Abstract: Low frequency arrays (120 to 400 MHz) can be calibrated using a sky model based on sky maps. For a single antenna, the variation of measured power with the Earth's rotation is needed to separate receiver gain and noise temperature. For an array, an instantaneous calibration can be made using normalized correlations or normalized beam power for which the receiver gain cancels out. Examples of the sky model for various antenna orientations are shown along with measured data used to perform a calibration. The effects of unmodeled mutual coupling of the received signals and receiver noise on the calibration method are analyzed and found to result in errors of only a few percent in an array of 5×5 crossed-dipoles designed to observe the deuterium line at 327 MHz. *INDEX TERMS*: 0604 Antenna arrays; Radio Science: Instrumentation and techniques; 6949 Radio Astronomy; *KEYWORDS*: Calibration, active antenna, antenna arrays, sky model.

1. Introduction

The conventional method of the calibration of radio astronomy antennas involves the use of a switch between the low-noise amplifier and the antenna port. The switch allows hot and cold loads to be connected to the amplifier in order to first calibrate the amplifier gain and noise temperature. Following the gain and noise calibration, the antenna temperature can be determined. The antenna temperature must then be converted to a sky brightness measurement, following a correction for the beam efficiency, which in turn can be measured using a strong radio source of known flux density. The complete calibration procedure is described by Pauliny-Toth and Shakeshaft [1962].

The calibration of an active antenna is difficult because there is no standard impedance interface between the antenna structure and the active low-noise amplifier since the amplifier and antenna elements form a single integrated structure. One method of calibration is to use a test source of known strength to measure the system gain. This test source can be a signal generator or calibrated noise diode connected to an antenna of known gain. One problem with using a test source is that it is difficult to position the source in the far field of the active antenna or array of active elements.

Another problem is that the entire system needs to be in an anechoic chamber of known temperature, [An, Nauwelaers, and Van de Capelle 1993] in order to determine the absolute antenna temperature. Outside of an anechoic chamber, the output of the active amplifier is the sum of the noise from the active device and the sky, plus ground noise convolved with the antenna gain pattern. In this case the test source can be used to measure system gain and noise but the amplifier noise cannot be separated from the sky and ground noise without a sky model. We use a sky model to calibrate the active antenna arrays for observation of the deuterium line at 327 MHz. One of these arrays, consisting of crossed-dipoles, is shown in Figure 1.

2. Sky brightness map

In order to generate an accurate sky model, we start with an all-sky map. The most complete map available is the map at 408 MHz made by Haslam et al. [1982]. The Haslam map was generated by combining sky surveys made with three different telescopes. The pixel size of this map is 0.3516º square. We checked the calibration of the 408 MHz map by comparing the flux density of Cassiopeia A against the value measured by Baars et al. [1977]. As the Baars et al. measurement was done with a beam of 2.2º×2.2º, we averaged several pixels around the position of Cassiopeia A in the Haslam et al. map to obtain a flux density in a 2.2º square area. Several such averages were computed and compared to the Baars et al. value. The maximum error obtained in this comparison was ~3%. In addition other points ($\alpha=2^h$, $\delta=60^\circ$; $\alpha=20^h$, $\delta=-30^\circ$) in the Haslam et al. map were compared with points in a 400 MHz map made by Taylor [1973]. The brightness temperatures agreed to within a few degrees. In this comparison, the Haslam et al. map was corrected to 400 MHz using a spectral index of 2.55 (Lawson et al. 1987).

To obtain a sky brightness map at 327 MHz we increased the temperatures in the 408 MHz map by a factor of 1.7582 which corresponds to a spectral index of 2.55.

3. Single antenna element

First, we describe the use of a sky model to measure the antenna and amplifier noise temperatures. For a single fixed active antenna element we need to observe the change in noise output power with sidereal time. In this case we need to rely on the active amplifier gain being stable, or use a test source to constantly calibrate the gain variation with time.

The output power is given by:

$$W\left(LST\right) = g\left(T_{sky} \otimes D + T_{amp}\right) \quad (1)$$

where W = output power vs Local Sidereal Time (LST) in units of Kelvin per Hz

g = gain

T_{sky} = sky and ground noise vs LST in coordinates of the antenna

D = antenna directivity

T_{amp} = amplifier noise temperature

$\otimes$ = convolution

4. Sky model for a single active element

Starting with each element in the sky map, we calculate all the needed coordinate transformations to place the element at a known location in coordinates fixed to the antenna. For example, to convert galactic latitude and longitude coordinates of the sky brightness map we perform the following steps:

1] Convert galactic latitude and longitude to right ascension and declination.

2] Convert right ascension and declination for a given local sidereal time to azimuth and elevation.

3] Convert azimuth and elevation to coordinates which are fixed to the antenna structure for which the beam pattern was measured or modeled.

The modeled antenna temperature is now obtained by convolution with the antenna beam pattern.

$$M(LST) = (1/p)\sum_i \sum_j T(i,j)\, D\big(\theta(i,j)\,\phi(i,j)\big)\cos\big(glat(i)\big) \quad (2)$$

where: *i, j* are indices of galactic latitude and longitude in the sky map

T(i,j) = sky map for elevation above the horizon and the effective ground temperature below horizon

D = antenna power pattern which is proportional to the directivity

θ = angle with respect to dipole direction

ϕ = elevation above the ground plane

$$p = \sum_i \sum_j D(\theta,\phi)\cos\big(glat(i)\big)$$

glat(*i*) = galactic latitude corresponding to map index *i*

For an active antenna consisting of a dipole a distance *h* wavelengths above a ground plane

$$D = \Big[\big(\cos\big((\pi/2)\cos\theta\big)\big/\sin\theta\big)\sin(2\pi h\sin\phi)\Big]^2 \quad (3)$$

The theoretical beam patterns for the E and H planes are shown in Figure 2. Figures 3 and 4 are examples of the sky model at 327 MHz computed for the 42.5º latitude of Haystack Observatory and for two cases of orientation normal to the ground plane. The dipole orientation is given as the angle relative to the azimuth, measured clockwise in the plane of the ground plane. To account for the trees that surround the area where measurements were made, a temperature of 290 K was assumed for all elevations below 5º. As an example of the use of the sky model, Figure 5 shows the best fit between measured power data and the sky model, plus the receiver noise temperature. The scale of the power data and an offset in the sky model to account for the receiver noise were adjusted to obtain the best weighted fit. Only nighttime data were weighted in the fitting process as the sun was quite active during this period. The best fit receiver noise was 25±5 K.

5. Calibration of an array

In the case of an array it is possible to make an instantaneous calibration with a sky model if both the beam power and the sum of the power from individual elements are available. In this case we need to assume that all elements are equal, and either ensure that cross-coupling effects are small or can be modeled.

If we use the ratio of the beam power to the sum of power from the individual active elements we obtain a quantity which is independent of gain and one which can be compared with a sky model to obtain an instantaneous estimate of the amplifier noise temperature. In this case the normalized beam model R is

$$R = \frac{T_{sky} \otimes \left| \sum_k \sum_\ell A e^{i\left(\theta_{k,\ell} - \phi_{k,\ell}\right)} \right|^2 + N T_{amp}}{N\left(T_{sky} \otimes |A|^2 + T_{amp}\right)} \tag{4}$$

where: A = complex voltage response of each element

$|A|^2$ = D directivity

T_{sky} = sky and ground noise temperature

$\theta_{k,\ell}$ = phase in the direction of the sky element for element with 2-D indices k, ℓ

$\phi_{k,\ell}$ = beam steering phase (zero for boresight)

N = number of elements

T_{amp} = amplifier noise temperature

The above relationship assumes:

1] All elements have the same response.

2] Mutual coupling between elements is negligible.

3] Each element receives a negligible coupled noise contribution from adjacent elements.

As an example, we show in Figures 6 and 7, from 18 to 6 hrs LST, the sky model with added receiver noise and the measured data for the normalized beam of the 5×5 array. The best fit to the receiver noise was 40±10 K. Again only the nighttime data were given any weight in the fit, although the sun was less active and lower in the sky in September. The differences between the measured and the model curves in figures 5,6 and 7 are likely due to several factors as follows:

1] Inaccuracies in the sky map. As discussed earlier, we compared maps from different authors and found good agreement so we expect the sky brightness to be in error by less than 3%.

2] Inaccuracies in the assumed antenna pattern. We assumed perfect dipole and ground plane in the analytic expression of equation (3). In the case of the single dipole in Figure 5 and the dipoles along the edges of the 5×5 array the ground plane only extends 0.8λ from the dipole. For this small ground plane the integrated back lobe is about -15 dB or 3%. Another source of antenna pattern error is in imperfect balance in the active dipoles which results in a common mode response to vertical polarization at a level of about -20 dB or 1%.

3] A major potential error source is the Sun. The individual dipoles have a maximum gain of 8 dBi so that the typical quiet Sun flux of 3×10^5 Jansky at 327 MHz results in an antenna temperature of up to 46 K. The effect of the Sun on the individual dipoles is significant, especially when the Sun is active and at a high elevation, and hence only nighttime data should be used for calibration.

4] Mutual coupling between dipoles in the array is major error source in the calibration of the array when using the normalized correlations or normalized beam power. The next section discusses the effects of mutual coupling in some detail.

6. Effects of mutual coupling between elements

The effects of mutual coupling can be incorporated into the normalized beam model using the following equation with added coupling terms:

$$R = \frac{T_{sky} \otimes B + NT_{amp}}{\left(T_{sky} \otimes C + NT_{amp}\right)} \tag{5}$$

where $$B = \left| \sum_k \sum_\ell c_{k,\ell} \right|^2 \tag{6}$$

$$C = \sum_k \sum_\ell \left| c_{k,\ell} \right|^2 \tag{7}$$

where $$c_{k,\ell} = Ae^{i\left(\theta_{k,\ell} - \phi_{k,\ell}\right)} + \sum_{m \neq k} \sum_{n \neq \ell} \alpha_{k-m,\ell-n} Ae^{\left(\theta_{m,n} - \phi_{m,n}\right)} \tag{8}$$

where $\alpha_{k-m,\ell-n}$ = complex coupling coefficient between elements k, ℓ, and m,n

N = number of elements.

To test the magnitude of the mutual coupling effects we used the 5×5 active dipole array being constructed to observe the 327 MHz line of deuterium. This array consists of half-wave crossed-dipole elements which are separated by 0.8 λ on a regular grid. The dipoles are approximately 0.2 λ above a ground plane. The results of measuring the mutual coupling between dipoles are as follows:

Configuration	Coupling dB	Geometry
Adjacent Dipoles	-24	0.8λ echelon
Diagonal dipoles	-26	1.13λ side by side
Diagonal dipoles	-30	1.13λ end to end

Owing to the difficulty of measuring the mutual coupling phases we used the method of moments (EZNEC [1981]) to model the 5×5 array, and the transistor scattering parameters to model the load on the antenna elements. Numerical simulations show that the voltage coupling coefficient is approximately proportional to the inverse distance squared. In free space the voltage coupling coefficient decreases with distance, but the presence of a ground plane results in a much more rapid decrease in distance, so the closest configurations listed in the table above dominate. To a large degree the effects average out over the entire array so that the beam power, average power of all dipoles, and the normalized beam power are only influenced at the level of a few percent. In Figure 8 we show the sky model for the normalized beam, and a 40 K receiver noise temperature with and without the estimated mutual coupling. Mutual coupling also results in an increase in system noise due to the coupling of the noise out of one amplifier into another. Consider the signals x and y from two active antennas. If we assume the temperature out of each active antenna is equal to the amplifier noise temperature and is perfectly correlated with the output noise then

$$x = n_x + \alpha n_y$$
$$y = n_y + \alpha n_x \qquad (9)$$

where: n_x, n_y are the amplifier noise, and

α = coupling coefficient

The added noise in each element is

$$\sum |\alpha|^2 \qquad (10)$$

where the sum is over all other elements. Given that the largest coupling is -24 dB, the added noise is only about 2% of the amplifier noise, or about 0.4 K for 20 K amplifiers. The added noise in the beam power can be estimated by the influence on each baseline. In this case:

$$\left\langle xy^* \right\rangle = \left\langle n_x n_y^* \right\rangle + \alpha^* \left\langle |n_x|^2 \right\rangle + \alpha \left\langle |n_y|^2 \right\rangle \qquad (11)$$

so that the correlations are biased by twice the real part of the coupling, or a maximum of about 12%. The effect of this bias is to generate an artificial source normal to the ground plane since the phases are zero. The overall effect is small when we consider that the amplifier noise is about 20 K and the short baselines are only about 20% of the total sum of baseline correlations in the beam. In this case the level is about 0.4 K. The added noise is estimated from the standard deviation of the cross-correlations.

Using the properties of Gaussian random variables and normalized power

$$\left\langle xy^* \right\rangle = 1 + \alpha^* + \alpha \qquad (12)$$

when $\left\langle |x^2| \right\rangle = \left\langle |y|^2 \right\rangle = 1$, and the noise in the correlation is: (13)

$$\left[\left\langle |xy^*|^2 \right\rangle - \left| \left\langle xy^* \right\rangle \right|^2 \right]^{1/2} = 1 + |\alpha|^2 \qquad (14)$$

so that the added noise from the mutual coupling from all elements to a given element is $\sum |\alpha|^2$.

The average fractional noise increase due to mutual coupling is $\frac{1}{N} \sum_{m \neq k} \sum_{\ell \neq n} |\alpha_{k-m,\ell-n}|^2$ or about 2% for the 5×5 array. From the above analysis we conclude that the effects of added noise from the mutual coupling is small.

The mutual coupling also results in a change in the gain or scaling of the beam power. The maximum magnitude of the scale change is approximately twice the real part of the cross-coupling averaged over all baselines or about 2.5%. This source of error could be eliminated by fitting the beam power to the sky model without normalizing by the sum of the power from each element. The combination of added noise and scale change in the beam power results in a difference of about 5% between the modeled and unmodeled effect of mutual coupling on the normalized beam ratio shown in Figure 8.

Conclusion

A sky model based on sky survey maps provides a practical means of calibrating low frequency antenna arrays to an accuracy of a few percent.

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Figure Captions

Figure 1. View, looking North, of a section of a 5×5 array of active antennas for observation of the deuterium line at 327 MHz. Trailer for RFI monitoring is in the background. The array is located at the MIT Haystack Observatory in Westford, Massachusetts.

Figure 2. The theoretical antenna power patterns for a half-wave dipole and 5×5 array of dipoles over a ground plane.

Figure 3. The sky brightness model for dipole at latitude 42.5º. The maximum response of the dipole, which is normal to the ground plane, is pointed at an azimuth of 120º and an elevation of 53º. The two curves are for the two slant polarizations of the dipole.

Figure 4. Sky brightness model for a horizontal orientation of the ground plane with the dipole maximum response at the zenith. An orientation of 135º corresponds to an azimuth measured in degrees east of north as shown in Figure 1 (upper dipoles).

Figure 5. The best fit of a sky model at 327 MHz to the total power data from a single dipole pointed at the zenith and oriented N-S. The data are consistent with an amplifier noise contribution of 25±5 K. The solid squares are the sky model plus 25 K. Data from 23 to 8 hours LST are corrupted by solar emission. These data was taken on 9 May 2002.

Figure 6. The best fit of a sky model at 327 MHz (solid curve) to the normalized beam of the 5×5 array which was pointed at the zenith. The sky model is plotted for an amplifier noise contribution of 40 K. The error in the fit is estimated to be ±10 K. The thin lines are the data for 3 days, 28, 29, 30 September 2003.

Figure 7. The best fit for the polarization perpendicular to that in Figure 6. The data (thin lines) are for 3 days; 28, 29, 30 September 2003.

Figure 8. The difference in sky models for the normalized beam power with and without the estimated effects of mutual coupling in the 5×5 array at 327 MHz. A 40 K amplifier noise contribution was assumed.

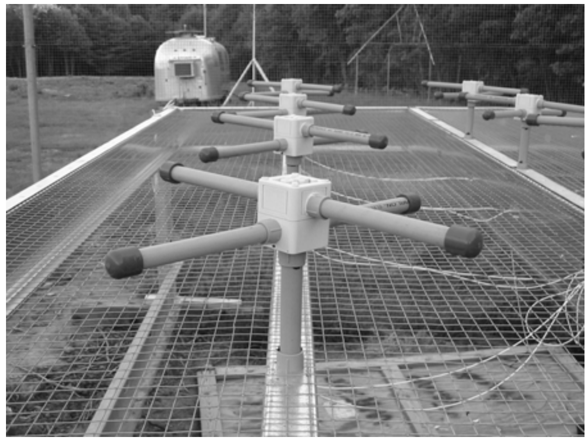

Figure 1

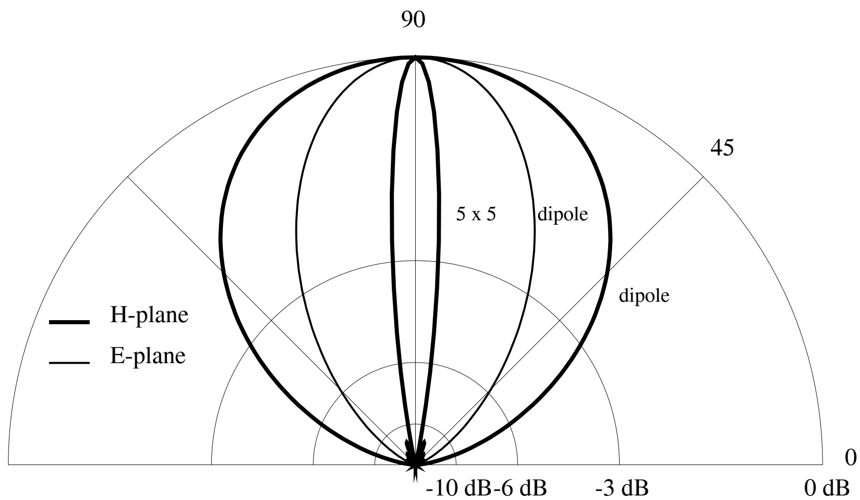

Figure 2

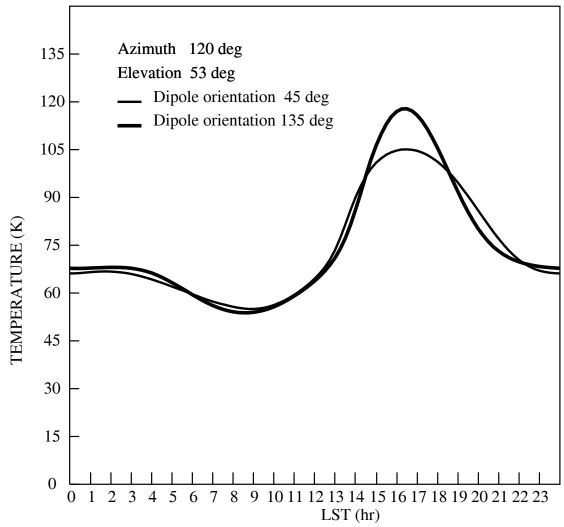

Figure 3

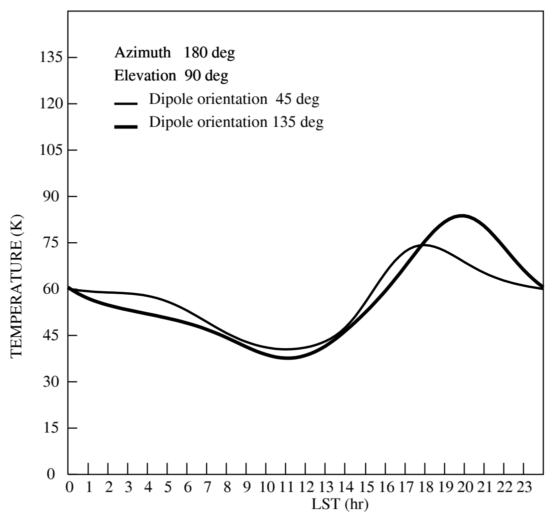

Figure 4

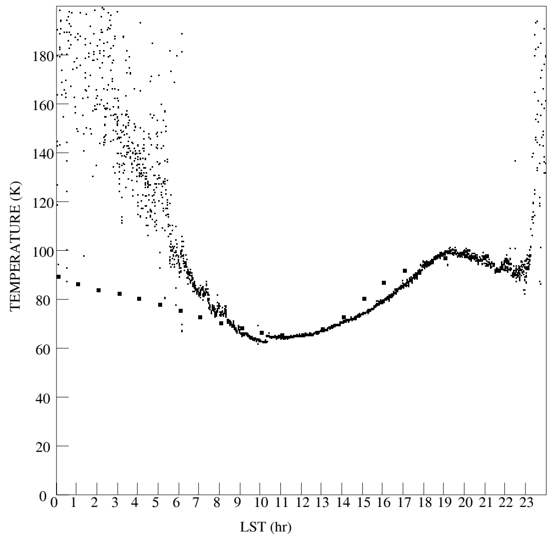

Figure 5

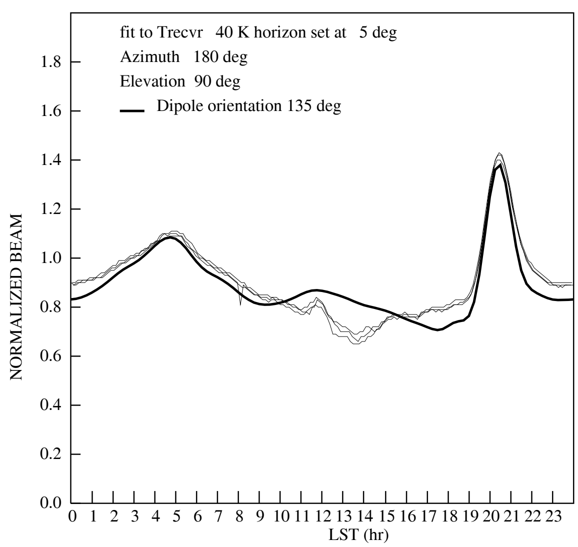

Figure 6

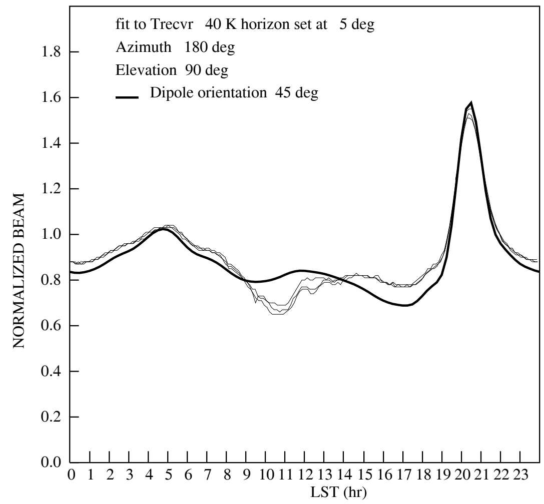

Figure 7

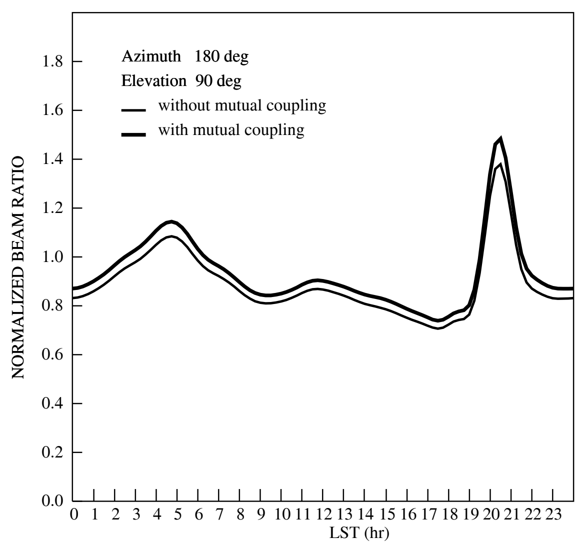

Figure 8