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To: LOFAR Group
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Subject: LOsim accuracy

1 LOsim and FFT

LOsim is the growing suite of software routines that will be used to simulate LOFAR performance. A key component of LOsim is a module that populates a sky brightness array with sources and performs an FFT to generate perfect complex visibilities on the (u,v) plane. Since LOFAR mapping requires dynamic ranges of 10^6, the visibilities generated by LOsim must be extremely accurate. Numerical errors in the double floating point precision FFT used in LOsim are $\sim 10^{-17}$ so for a 10^6 dynamic range, we expect that the relative error in the visibility magnitude will be $\sim 10^{-11}$. This memo gives the results of comparing LOsim outputs with theoretical visibility values and concludes that the FFT in LOsim is delivering the expected accuracy.

2 Tests

The output of LOsim was checked against theoretical values for the case of a circular Gaussian brightness distribution offset from the center of the field of view. The particulars of the simulation are :

Field of View size (S):	1000 x 1000 arcseconds
Field of View array (M):	2048 x 2048
Brightness array (padded) (N):	2^{13}x2^{13}
(u,v) plane array (N):	2^{13}x2^{13}
(u,v) resolution:	$(M-1)/(NS) = 51.541\ \lambda$
Image Resolution :	$S(M-1) = 0.4885$ arcseconds
(u,v) extent :	$(M-1)/(S) = 2.111120 \times 10^5\ \lambda$
Source size:	5 x 5 arcseconds (FWHM)
Source offset:	(1.0,0.5) arcseconds
Source Flux Density:	1.0 Jy

The theoretical complex visibilities for a circular Gaussian offset from the map center is given by:

$$\mathcal{V}(u,v) = S_\circ \exp\left(-\pi^2 B^2 (u^2+v^2)/(4\ln 2)\right) \exp\left(2\pi i(ux_\circ + vy_\circ)\right) \quad (1)$$

where $S_\circ$ is the total source flux density, B is the FWHM size (radians), $(x_\circ, y_\circ)$ is the source offset from the map center (radians); (u,v) are measured in wavelengths. Corresponding visibility magnitudes generated by the FFT followed the theoretical values closely until a level of 10^{-17} was reached, at which point the FFT magnitudes started oscillating about this noise floor. Let's define the relative error in the visibility magnitude as computed by the FFT in LOsim as:

$$\text{Relative Error} = \frac{|\mathcal{V}_{\text{ls}} - \mathcal{V}|}{\mathcal{V}} \quad (2)$$

where $\mathcal{V}_{\text{ls}}$ is the visibility generated by LOsim. If the precision of the FFT is at the $\sim 10^{-17}$ level, then the Relative Error can be expressed approximately as:

$$\text{Relative Error} \simeq \frac{10^{-17}}{\mathcal{V}} \quad (3)$$

The relative error in magnitude was computed for 1000 randomly selected (u,v) points and is shown in Fig. 1 as a function of visibility magnitude. As expected, the error is approximated by Eq. 3 and the error at a map dynamic range of 10^6 is $\sim 10^{-11}$. Figure 2 shows the difference in visibility phase between the LOsim values and Eq. 1.

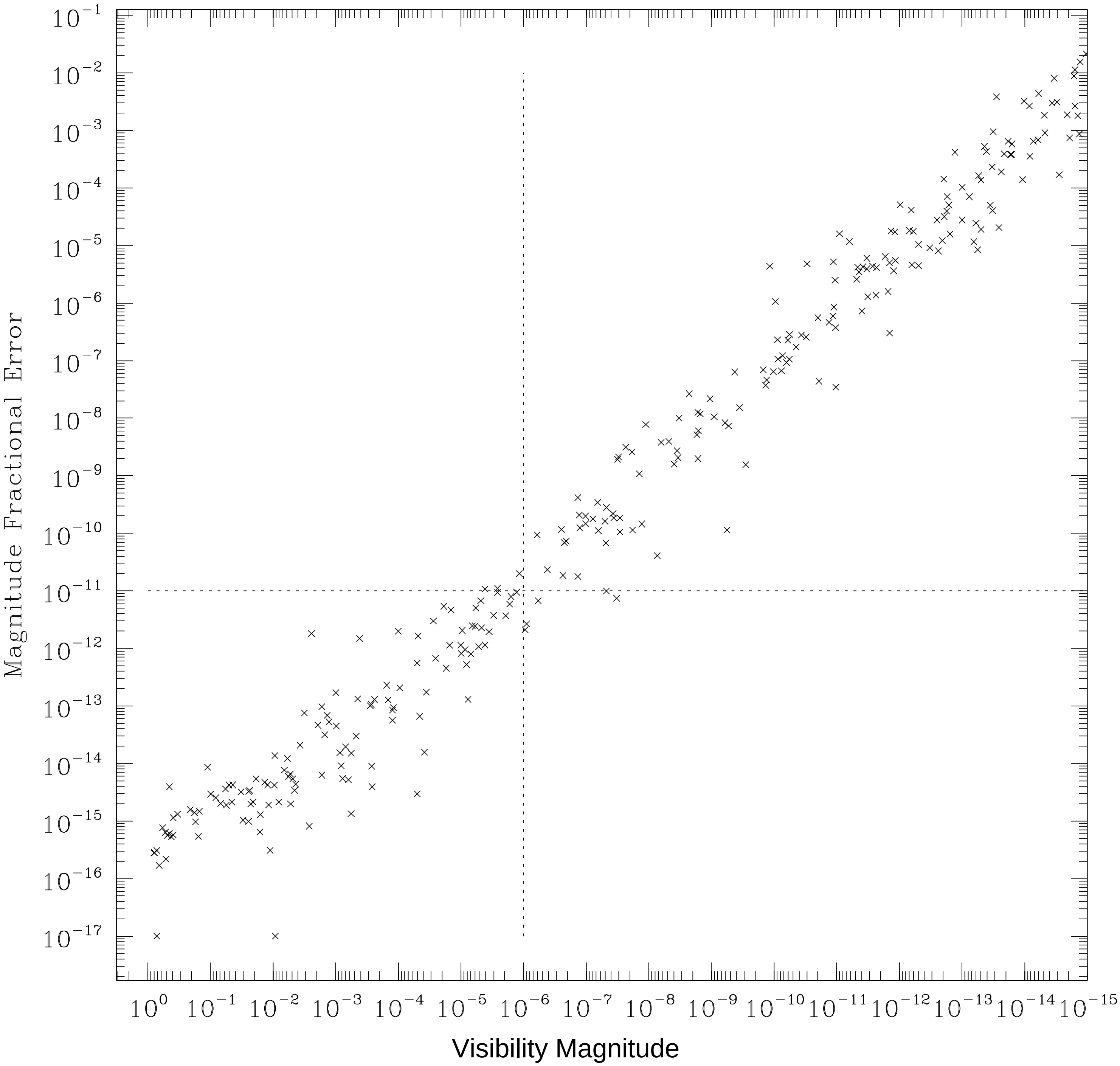

Figure 1: Relative visibility magnitude error as a function of visibility magnitude for 1000 randomly selected points in the (u, v) plane.

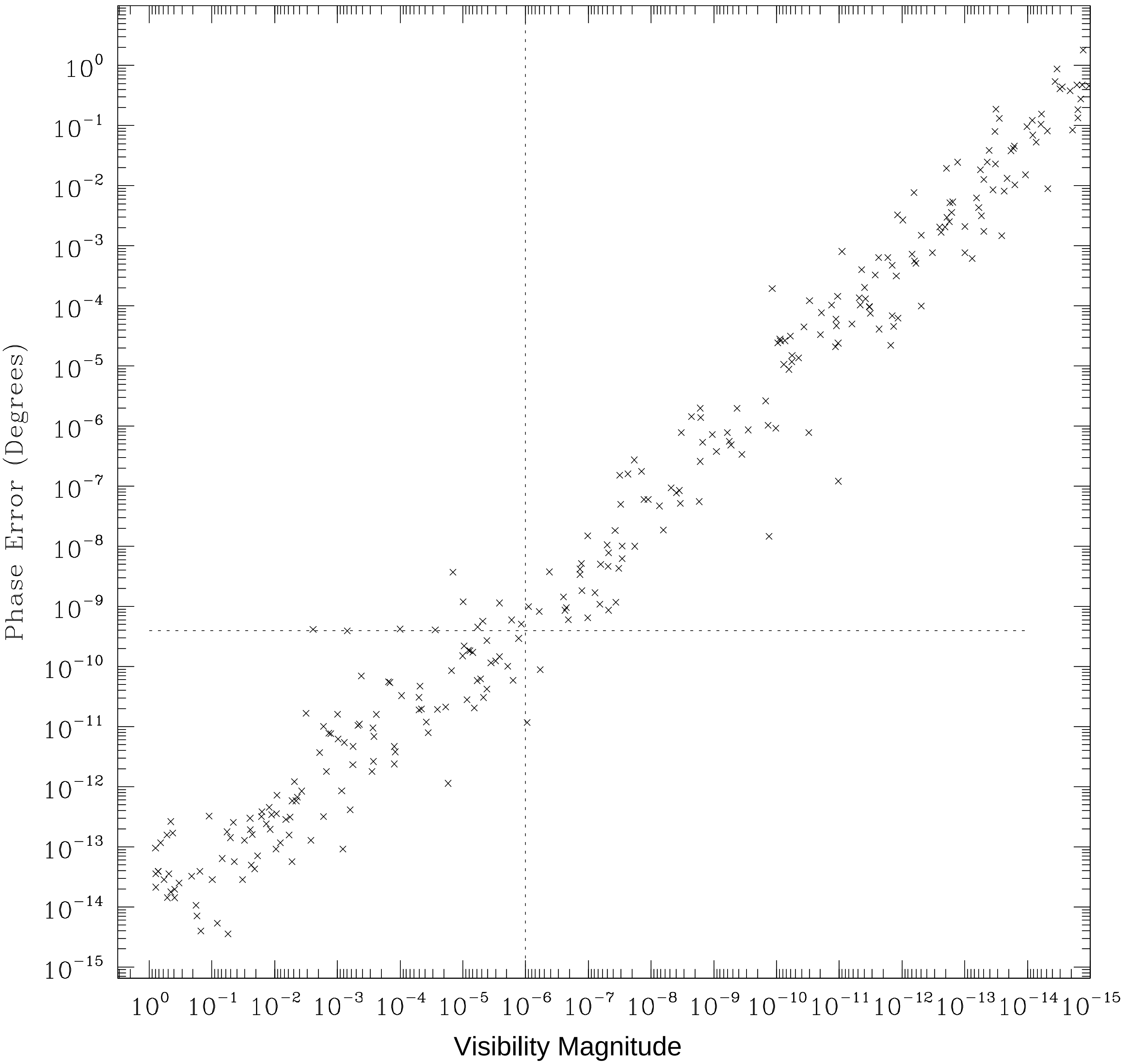

Figure 2: Phase error as a function of visibility magnitude for 1000 randomly selected points in the (u, v) plane.